



STUDIENZENTRUM GERZENSEE  
STIFTUNG DER SCHWEIZERISCHEN NATIONALBANK

## Program for Beginning Doctoral Students in Economics 1997

### Exam in Microeconomics

Monday, March 9, 1998, 14.00 - 17.00

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the expectations of PC's.
2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.
3. Good luck!

### CALVIN AND HOBBS



## 1. Control of a Self-Managed Firm

Consider a firm that has a production function given by

$$y = \theta \ell^{1/2} \quad \theta > 0,$$

where  $\ell$  is the number of workers (considered to be a continuous variable) and  $\theta$  is a parameter known to the firm. Impose on this firm a fixed cost  $A$  expressed in units of the numéraire. Let  $p$  be the price per unit of the good produced competitively by the firm (good 1).

- (a) Assume that the firm is managed by the workers and that its objective function is

$$\frac{p y - A}{\ell} = \frac{p \theta \ell^{1/2} - A}{\ell}.$$

Determine the optimal size (for the workers) of the self-managed firm.

Let  $w$  be the wage rate in the rest of the economy which is assumed to be competitive so that  $w$  represents the opportunity cost of labor in this economy. Why is the allocation of labor not optimal in general? What happens if  $w$  is too large?

- (b) Assume first that the government knows  $\theta$ . Consider the case in which  $w$  is small enough to justify the presence of a self-managed firm. Compute the tax per unit  $\tau$  on good 1 that restores an optimal allocation of labor. Show that we could also use a lump-sum tax  $T$  on the revenue of the self-managed firm to achieve the same objective. (Assume that the self-managed firm is of negligible size with respect to the rest of the economy).

- (c) Suppose now that the government does not know  $\theta$ . He uses a regulatory mechanism  $\ell(\tilde{\theta}), t(\tilde{\theta})$  which associates to an announcement  $\tilde{\theta}$  a labor input  $\ell(\tilde{\theta})$  and a transfer  $t(\tilde{\theta})$  to the labor managed firm. The firm's objective is now:

$$\frac{p\theta(\ell(\tilde{\theta}))^{1/2} + t(\tilde{\theta}) - A}{\ell(\tilde{\theta})}.$$

Characterize the regulatory mechanisms which induce truthful revelation of  $\theta$ . (First order and second order conditions)

Suppose that the government wishes to maximize the expectation of  $p\theta\ell^{1/2} - w\ell$ . Show that the solution of this maximization is not implementable in view of the incentive constraints.

- (d) Extend the analysis to the case of a self-managed monopolist who is facing an inverse demand function  $p = a - by$ .

2. Take the Rotemberg-Saloner model with, for each period,  $\text{Prob}(q = 4 - p) = 1/3$  and  $\text{Prob}(q = 2 - p) = 2/3$ . Assume two firms in the market, with constant marginal cost = 1, and prices as strategic variables.
- (a) What is the minimum discount factor  $\delta$  for which the monopoly outcome is sustainable for each demand realization ?

- (b) For which  $\delta$  can one sustain  $p = 1$  if  $q = 4 - p$  and  $p = 3/2$  if  $q = 2 - p$ ? Compare with (a).
- (c) Explain the economic forces that lead to opposite predictions concerning the cyclical behavior of markups in the Rotemberg-Saloner and Green-Porter models.

3. Assume a bargaining model where a seller of a good (for which he has 0 valuation) can make offers to a potential buyer whose valuation of the good  $b \in \{\underline{b}, \bar{b}\}$ , where  $\underline{b} < \bar{b} < 2\underline{b}$ . The buyer knows her valuation. It is common knowledge that the seller's prior belief is  $\text{Pr ob}(b = \underline{b}) = 1/2$ .
- (a) What is the perfect-Bayesian equilibrium of the game if it only involves a single take-it-or-leave-it offer by the seller (and a zero payoff for both players in case of rejection of this offer by the buyer) ?

- (b) What is the perfect-Bayesian equilibrium of the game if, after rejection of his offer, the seller can, in a second period, make a new, final offer, and if discount factors between periods are  $\delta_B$  and  $\delta_S$  for the buyer and seller respectively? In particular, will trade take place immediately with probability 1 if  $\delta_B = \delta_S$ ? And if  $\delta_B \neq \delta_S$ ?

(Optional)

4. Consider an indirect utility function of the form

$$u(w,p) = a(p) + b(p)w,$$

where  $w$  is wealth and  $p$  is a price vector.

(a) What characteristics must the functions  $a(p)$  and  $b(p)$  display?

(b) Show that the wealth elasticity of demand for the  $k$ th good is constant.

- (c) Graph the income expansion paths, i.e., Engel curves.

- (d) If preferences are homothetic, what additional restriction would be placed on the indirect utility function?

(Optional)

5. Consider a profit-maximizing firm that is producing a single output,  $y$ , using a vector of  $K-1$  inputs,  $x$ , with a strictly concave production function. The firm is a price taker in all markets. The government has imposed specific taxes on various commodities, so that the gross of tax price vector is  $P = (p_1 + t_1, \dots, p_K + t_K)$ . Use the envelope theorem to derive the effect on profits, i.e., the maximized value of profits, of a small increase in the tax on the  $k$ th input.

(Optional)

6. Give an example of a consumer for which the excess demand function is well defined for every non-zero, non-negative vector of prices. Make sure that preferences be continuous, strictly convex and locally non-satiated.
  
7. Consider an exchange economy. State precisely a set of sufficient conditions on preferences and endowments implying the existence of price equilibrium. Make sure that these conditions allow for the possibility of some prices being zero.