STUDIENZENTRUM GERZENSEE
STIFTUNG DER SCHWEIZERISCHEN NATIONALBANK

Swiss Program for Beginning Doctoral Students in Economics 1998

Final Exam in Microeconomics

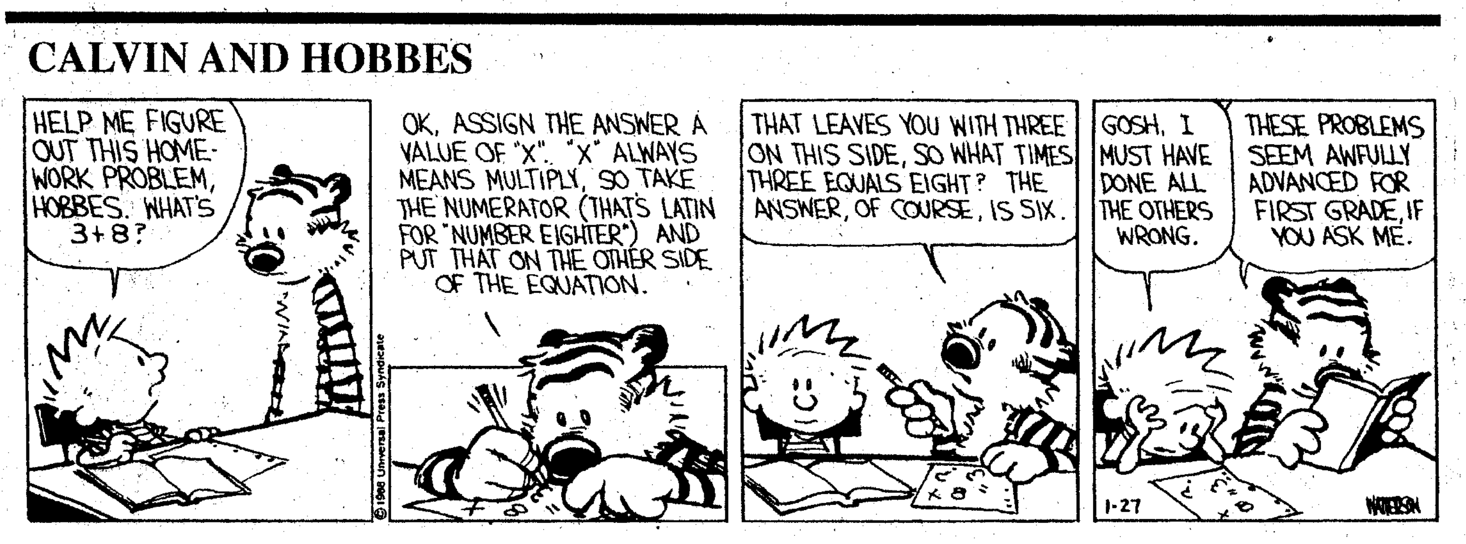
Wednesday, March 10, 1999, 08.30 - 11.30

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.

2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.

3. Good luck!

ID-Number:____



Part I (20 points)

1. Let f(x) be a production function that is homogeneous of degree one. If all inputs are strictly positive at prevailing prices so that $w_i = p\delta f(x)/\delta x_i$, show that profits must be 0.

2. Consider a utility maximizing consumer who chooses two goods at prices (p_1,p_2). The substitution matrix is:

$$\begin{matrix} \delta h_1(p,u)/\delta p_1 & \delta h_1(p,u)/\delta p_2 \\ \delta h_2(p,u)/\delta p_1 & \delta h_2(p,u)/\delta p_2 \end{matrix} = \begin{matrix} a & b \\ c & d \end{matrix}$$

Suppose that you are given a. How can you determine b, c, and d?

3. Suppose that an individual has a Bernoulli utility function $u(m) = m^{1/2}$.

a. What is the Arrow-Pratt coefficient of absolute risk aversion?

b. Is the Arrow-Pratt coefficient of absolute risk aversion increasing or decreasing in wealth, i.e., income?

c. Calculate the certainty equivalent of a lottery (9,4,1/2,1/2), i.e., a lottery that pays 9 with probability ½ and pays 4 with probability ½.

Part II (20 points)

Comment on the role of convexity for the fundational theory of welfare Economics.

Try to be precise and give some examples and counterexamples.

Part III (70 points)

Question 1

Consider a Cournot duopoly problem where demand is given by: $p=3-q_1-q_2$ and where the constant marginal costs are $c_1=c_2=1$.

Assume firm 1 can spend a fixed cost F to raise firm 2's marginal cost to 2.5 (example: lobby to impose a tariff if firm 2 is a foreign firm).

(a) what is the level of F above which it is not profitable for firm 1 to spend this fixed cost ?

(b) explain in words how firm 1's reasoning would change if firms competed in prices instead of quantities. Is the difference the same as when F serves to decrease firm 1's marginal cost ?

Question 2

Spence model with quadratic education cost and productive education.

Assume two types of workers, H and L, with probabilities a and 1-a in the population.

Their respective utilities are $w-c_He^2$ and $w-c_Le^2$, where e is the level of education, w the wage and we have $0<c_H<c_L$.

Worker productivity is L+e for type L and H+e for type H (with $0<L<H$).

(a) show graphically the equilibrium if worker types are publicly observable.

(b) show graphically when the absence of public observability of worker types matters.

(c) in the case where things differ between (a) and (b), describe the sets of separating and pooling Bayesian-perfect equilibria (be precise).

Part IV (70 points)

Question

There is a continuum of risk neutral borrowers with no personal wealth and limited liability. A proportion Π of borrowers, the good type, have sure projects with return h and a proportion $1 - \Pi$, the bad type, have (stochastically independent) projects with return h with probability $p < 1$ and return 0 with probability $1 - p$. All borrowers have outside opportunities valued at $u > 0$ and the type of a borrower is his private (non verifiable) information.

There is a single bank available for loans which has a refinancing rate of r. The bank offers contracts to maximize its expected profit. For simplicity, we assume that all projects which require one unit of investment are socially valuable, i.e. $ph > r + u$ or $h > \frac{r+u}{p}$.

Let us refer to the good type with the index 1 and to the bad type with the index 2. From the revelation principle, we know that any individual lending strategy is equivalent to a revelation mechanism $(r_1, P_2), (r_2, P_2)$ where P_i is the probability of obtaining a loan and r_i is the interest rate to be paid to the bank if the borrower announces that he is of type i.

The bank maximizes its expected profit under the incentive and participation constraints of the representative borrower. Explain why the maximization program of the bank writes:

$$\max \Pi P_1(r_1 - r) + (1 - \Pi) P_2(pr_2 - r)$$

s.t.

$$\begin{aligned} P_1(h - r_1) &\geq P_2(h - r_2) & (1)\\ pP_2(h - r_2) &\geq pP_1(h - r_1) & (2)\\ P_1(h - r_1) &\geq u & (3)\\ pP_2(h - r_2) &\geq u. & (4) \end{aligned}$$

1- Characterize the optimal lending contracts chosen by the bank.

2- One may wonder if group lending may be a more powerful instrument for the bank. Let us restrict the analysis to groups of two borrowers. Borrowers do not know each other. They know that the matching with other borrowers will be random. We will have to distinguish two cases according to the composition of the set of borrowers applying for loans.

Consider first the case where everybody applies. For a good type to apply requires

$$\Pi(h - r_0) + (1 - \Pi)(p(h - r_0) + (1 - p)(h - x)) \geq u \tag{5}$$

where r_0 is the payment if both partners are successful and x is the payment that a succesful partner must make when his partner fails.

For a bad type to apply also requires:

$$\Pi p(h - r_0) + (1 - \Pi)(p^2(h - r_0) + p(1 - p)(h - x)) \geq u \tag{6}$$

or

$$\Pi(h - r_0) + (1 - \Pi)(p(h - r_0) + (1 - p)(h - x)) \geq \frac{u}{p}. \tag{7}$$

Explain these constraints, write the new bank's maximization problem: Characterize the optimal group lending contracts. Discuss.

3- Extend the analysis to the case where the borrowers know each other.