

**Swiss Program for Beginning Doctoral Students in Economics 2000**

**Exam in Macroeconomics**

**Friday, July 28, 2000, 14.00h - 16.00h**

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.
2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously. Please use **a pen** rather than a pencil so that your answers can be read without problems.
3. Good luck!

ID-Number: \_\_\_\_\_

# Studienzentrum Gerzensee

Macroeconomics Doctoral Course  
Midterm Exam

July, 2000

## 1. Short Answer Questions

(20 points)

- (a) True, false or uncertain: Global stability implies local stability.

(b) True, false or uncertain: An overlapping generations economy converges to a steady state level of the capital stock that is always dynamically efficient.

(c) True, false or uncertain: If technical progress is costly to produce it cannot emerge in an economy where both goods and factor markets are perfectly competitive.

(d) Suppose that a stock price  $p$  is determined by

$$p_t = \beta[E_t p_{t+1} + d_{t+1}]$$

with  $\beta < 1$  being a market discount factor and with dividends obeying

$$d_t = g d_{t-1}.$$

What is the interpretation of  $g$ ? Under what condition is the stock price finite, if we assume that it is the present discounted value of future dividends?

## 2. Demand for Materials

(20 points)

Suppose that a firm seeks to maximize its present discounted value at the market discount factor  $\beta = \frac{1}{1+r}$ , where  $r$  is the real interest rate. Suppose also that this firm has the following (decreasing returns to scale) production function

$$y_t = f(x_{t-1})$$

where  $y_t$  is output and  $x_{t-1}$  is the quantity of materials that were purchased at date  $t-1$ . Suppose finally that the price of materials is perfectly predictable and is called  $p_t$ .

- (a) Define the firm's flow of profits at each date  $t$  and its present discounted value;

(b) Find the efficiency condition which determines the firm's optimal choice of materials input.

(c) Supposing that  $f(x_{t-1}) = a(x_{t-1})^\alpha$ , find the firm's demand for materials at each date;

(d) How is this demand related to the neoclassical theory of investment?

### 3. Dynamic Programming

(20 points)

Consider a household that has the objective

$$U = \sum_{t=0}^2 \beta^t \frac{1}{1-\sigma} (c_t + \theta c_{t-1})^{1-\sigma}$$

where  $1 > \theta > 0$ . Suppose that this household has wealth  $a_0$  in period 0 and has prior consumption of  $c_{-1}$ . Suppose finally that the household's wealth evolves according to

$$a_t = (1+r)(a_{t-1} - c_t)$$

(a) Describe how this objective differs from the standard time separable objective;

(b) Set up the household's optimal consumption problem as a dynamic programming problem. Call the value functions  $v_0$ ,  $v_1$  and  $v_2$ .

(c) Solve for optimal  $c_2$  and determine  $v_2$ .

(d) Solve for optimal  $c_1$  and determine  $v_1$ .

(e) Solve for optimal  $c_0$  and determine  $v_0$ .

(f) Does this model imply that the growth rate of consumption depends only on  $\beta, r, \sigma$ ?

#### 4. The Neoclassical Model

(20 points)

Consider the following version of the neoclassical growth model:

$$\begin{aligned}U &= \sum_{t=0}^{\infty} \beta^t \log(C_t) \\Y_t &= A_t K_t^\alpha N^{1-\alpha} \\K_{t+1} &= (1 - \delta) K_t + I_t \\(1 - \tau)Y_t &= C_t + I_t\end{aligned}$$

where  $\tau$  is the tax rate on output.

(a) Compute the steady-state levels of output, consumption, investment, and capital.

(b) What is the condition that needs to be imposed to guarantee that utility is finite? Why is it important to impose this condition?

(c) Describe the steady-state effects of an increase in the tax rate  $\tau$  on the after-tax real interest rate. Provide an intuitive explanation for the results.

(d) Suppose that  $\delta = 1$ . Show that there is an analytical solution to this model. Use this solution to show that the economy is globally stable.

## 5. Solve Only One of the Following Two Questions

(20 points)

### 5.1. Small Open Economy

Consider the following simple model of a small open economy in continuous time. Each agent in the economy seeks to maximize her expected utility defined as:

$$U = \int_0^{\infty} e^{-\rho t} \frac{C_t^{1-\sigma}}{1-\sigma} dt.$$

Production of the single good obeys:

$$Y_t = A_t K_t^\alpha N^{1-\alpha}.$$

This economy can borrow or lend in the international capital market. The trade balance represents the difference between domestic output and domestic expenditure:

$$Y_t = C_t + I_t + TB_t.$$

When the economy has a trade surplus it accumulates net foreign assets ( $a_t$ ), while a deficit is associated with a decline in net foreign assets. The return on these assets equals the exogenous rate  $r^*$ , so the evolution of  $a_t$  is given by:

$$\begin{aligned} \dot{a}_t &= r^* a_t + TB_t. \\ \lim_{t \rightarrow \infty} e^{-r^* t} a_t &= 0 \end{aligned}$$

To simplify assume that  $r^* = \rho$ . The capital stock follows the usual accumulation equation:

$$\dot{K}_t = I_t - \delta K_t.$$

(a) What is the meaning of the condition  $\lim_{t \rightarrow \infty} e^{-r^* t} a_t = 0$ ?

(b) Characterize the steady state of this economy.

(c) Describe the effects of an increase in  $a_0$  on the trade balance of this economy.

## 5.2. Monopolistic Competition

Consider the following model in which final output producers use a range  $N$  of intermediate inputs  $x_i$ , which they combine according to the following production function:

$$Y = \left[ \int_0^N x_i^\nu di \right]^{1/\nu}.$$

Each intermediate good is produced with labor ( $L_i$ ) according to the following production function:

$$x_i = AL_i - \phi.$$

where  $\phi$  is a overhead cost required to operate the firm.

(a) Solve the output producer problem, determining the demand for good  $x_i$ .

(b) Compute the price chosen by each monopolist as a function of the wage rate.

(c) Assume that there is free entry and exit. What is the equilibrium number of firms?