STUDIENZENTRUM GERZENSEE
STIFTUNG DER SCHWEIZERISCHEN NATIONALBANK

Swiss Program for Beginning Doctoral Students in Economics 2000

Exam in Microeconomics

Saturday, July 29, 2000, 09.00h - 11.00h

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.

2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously. Please use **a pen** rather than a pencil so that your answers can be read without problems.

3. Good luck!

ID-Number:____

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1. (20 points) Consider a consumer who consumes only two goods and satisfies Walras' Law. When prices are $(2, 4)$ he demands $(5, 10)$. When prices are $(6, 3)$, he demands $(10, y)$. Nothing else of significance has changed between the two situations.

   (a) Suppose that $y = 5$. Do these consumption plans satisfy the weak axiom of revealed preference.

(b) For which range of y do these consumption plans violate the weak axiom?

2. (20 points) Consumer 1's expenditure function $e_1(p_1, p_2, u_1) = u_1\sqrt{p_1 p_2}$ and consumer 2 has utility function $u_2(x_1, x_2) = 43x_1^3 x_2^a$.

   (a) What are the Marshallian (market) demand functions for each of the goods by each of the consumers? Denote the wealth of consumer 1 by w_1 and the wealth of consumer 2 by w_2.

(b) For what value(s) of the parameter a will there exist an aggregate demand function that is independent of the distribution of income but only depends on $w_1 + w_2$?

3. (20 points) Let x be a vector of inputs and w the corresponding vector of input prices. Show: If the production function $f(x)$ has constant returns to scale, then the corresponding cost function can then be written as $c(w, y) = y \cdot c(w)$, where $c(w) = c(w, 1)$. Furthermore, the conditional input demand function $x(w, y)$ can be written as $x(w, y) = y \cdot x(w)$.

4. (30 points) In an exchange economy with I consumers, an allocation $\{x_1, x_2, ..., x_I\}$ is called "envy-free" if every consumer prefers his bundle to anyone else's.

- Show that , if initial endowments are equally shared ($\omega_i = \frac{\omega}{I}$ for all i) then any competitive equilibrium is envy free.

- Conversely, prove that if all consumers have the same utility function, then all envy free Pareto optimal allocation correspond to a competitive equilibrium with equal sharing of initial endowments.

- Represent the set of envy-free allocations in an Edgeworth box where consumers have Cobb-Douglas utilities

5. (30 points) Consider an exchange economy with one good (say, wheat), and S states of nature $(s = 1, ..., S)$, with respective probabilities π_s. There are I consumers $(i = 1, ..., I)$ characterized by preferences:

$$U^i(x^i) = \sum \pi_s u^i(x^i_s)$$

- Recall the precise characterization of Pareto optima in terms of marginal rates of substitution

- Apply this characterization to determine the Pareto optima of this economy in the particular case where

$$u^i(x^i_s) = (x^i_s)^{\alpha_i} \text{ for all i, with } \alpha_i \in [0, 1]$$

- same question when $u^i(x_s^i) = -\exp -\rho_i x_s^i$