STUDIENZENTRUM GERZENSEE
STIFTUNG DER SCHWEIZERISCHEN NATIONALBANK

Swiss Program for Beginning Doctoral Students in Economics 2001

Midterm Exam in Macroeconomics

Friday, July 27, 2001, 14.00h - 16.00h

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.
2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously. Please use **a pen** rather than a pencil so that your answers can be read without problems.
3. Good luck!

ID-Number:____

Studienzentrum Gerzensee

Macroeconomics Doctoral Course
Midterm Exam

July, 2001

1. Short Questions

(25 points)

(a) Suppose that an automobile costs P dollars new, the interest rate is r per year, the future resale value of a one year old automobile is $P\prime$ dollars, and there are maintenance expenses of D dollars that must be paid at the end of the year. What is the implicit rental price of the automobile for one year's time?

(b-1) If permanent income is defined as the annuity value of expected income, then why is it governed by the formula

$$y_{pt} = \frac{r}{1+r} E_t \sum_{j=0}^{\infty} (\frac{1}{1+r})^j y_{t+j}?$$

(b-2) If income is governed by the equation, $y_t = \theta_1 y_{t-1} + \theta_2 y_{t-2} + e_t$ then show that permanent income depends only on y_t and y_{t-1}. Also develop an equation that indicates how it depends on these variables.

(c) Use a two period model of labor supply, derived from maximizing

$$u(c_1, n_1) + \beta u(c_2, n_2),$$

subject to

$$c_1 + \frac{1}{1+r} c_2 = a + w_1 n_1 + \frac{1}{1+r} w_2 n_2,$$

to show that a higher growth rate of wages (w_2/w_1) leads to a higher growth rate of (n_2/n_1) labor supply. Discuss how this result relates to MaCurdy's work on life-cycle labor supply.

(d) Consider the following rational expectations model with two variables: a predetermined variable y_{1t} and a nonpredetermined variable y_{2t}. Does it have a unique, stable rational expectations solution, according to the criteria of Blanchard and Kahn?

$$E_t \begin{bmatrix} y_{1,t+1} \\ y_{2,t+1} \end{bmatrix} = \begin{bmatrix} 1.5 & 0 \\ 0 & .4 \end{bmatrix} \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}.$$

If so, why? If not, which of their conditions is violated?

2. Departures from time-separable preferences and dynamic programming

(25 points)

Consider a household that derives a flow of utility from consumption c_t in the following manner:

$$u(c_t, c_{t-1}) = \frac{1}{1-\sigma}[c_t + \theta c_{t-1}]^{1-\sigma},$$

with θ being a parameter that indexes how important prior period consumption is for current utility. Assume that the household has the utility objective:

$$U_t = \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, c_{t+j-1}).$$

Assume also that the household has a conventional flow budget constraint of the form

$$pa_{t+1} = y + a_t - c_t,$$

where p is the price of future wealth a_{t+1} in terms of current wealth a_t and y is income.

(a) Explain intuitively why c_{t-1} is a state variable for the household.

(b) If we define the variable, $k_t = c_{t-1}$, explain why the Bellman equation is

$$V(k_t, a_t) = \max_{c_t}\{u(c_t, k_t) + \beta V(k_{t+1}, a_{t+1})\},$$

where the maximization is subject to the pair of constraints

$$\begin{aligned} y + a_t - pa_{t+1} - c_t &= 0, \\ k_{t+1} - c_t &= 0. \end{aligned}$$

(c) Form a Lagrangian with the multiplier λ_t attached to the first of these constraints and θ_t attached to the second of these constraints, Find the first order conditions for c_t, k_{t+1}, a_{t+1}.

(d) Use the envelope theorem to determine $\partial V(k_t, a_t)/\partial k_t$ and $\partial V(k_t, a_t)/\partial a_t$.

3. The Neoclassical Model

(25 points)

Consider the following version of the neoclassical growth model:

$$\begin{aligned} U &= \sum_{t=0}^{\infty} \beta^t \log(C_t), \\ Y_t &= AK_t^{\alpha} N^{1-\alpha}, \\ K_{t+1} &= (1-\delta) K_t + I_t, \\ Y_t &= C_t(1+\tau^c) + I_t(1+\tau^i), \end{aligned}$$

where τ^c and τ^i are the tax rates on consumption and investment, respectively. To simplify assume that the tax revenue is used by the government to finance spending that does not yield utility or enhance the productivity of the representative agent.

(a) Compute the steady-state levels of output, consumption, investment, and capital.

(b) Describe the effect of an increase in the tax rate τ^i on the steady-state real interest rate. Provide an intuitive explanation for this result.

(c) Explain the effects of an increase in τ^c on the steady state level of output, investment and consumption.

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4. Solve Only One of the Following Two Questions

(25 points)

4.1. Monopolistic Competition

Consider the following model in which final output producers use a range of intermediate inputs x_i, indexed on the interval [0 1], which they combine according to the following production function:

$$Y = \left[\int_0^1 x_i^{\nu} di\right]^{1/\nu},$$

where $\nu < 1$.

Each intermediate good is produced with labor (L_i) according to the following production function:

$$x_i = L_i.$$

(a) Solve the output producer problem, determining the demand for good x_i.

(b) Compute the price chosen by each monopolist as a function of the wage rate.

(c) Compute the equilibrium profit made by each producer.

4.2. Small Open Economy

Consider the following model of a small open economy in continuous time. The economy is populated by a representative agent that seeks to maximize his lifetime utility which depends on the path for the consumption of tradables (C_t^T) and non-tradable (C_t^{NT}) goods:

$$U = \int_0^\infty e^{-\rho t}[\log(C_t^T) + \theta \log(C_t^{NT})]dt.$$

Each agent supplies N units of labor that can be allocated to the production of tradables (N_t^T) or of non-tradables (N_t^{NT}):

$$N = N_t^T + N_t^{NT}.$$

Production in the tradable sector is given by:

$$Y_t^T = A^T K_t^{1-\alpha}(N_t^T)^\alpha.$$

where K_t represents the stock of capital. The capital stock evolves according to:

$$\dot{K}_t = I_t - \delta K_t.$$

where I_t represents the investment flow.

Production in the non-tradables sector is given by:

$$C_t^{NT} = A^{NT} T^{1-\gamma}(N_t^{NT})^\gamma,$$

where T represents a fixed amount of land.

This economy can borrow or lend in the international capital market at rate r. The economy's flow budget constraint is given by:

$$\dot{a}_t = r a_t + (Y_t^T - C_t^T - I_t),$$
$$\lim_{t\to\infty} e^{-rt} a_t = 0,$$

where a_t represents net foreign assets. To simplify assume that $r = \rho$.

Characterize the steady state of this economy.