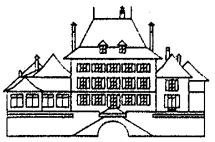

STUDIENZENTRUM GERZENSEE
STIFTUNG DER SCHWEIZERISCHEN NATIONALBANK

Swiss Program for Beginning Doctoral Students in Economics 2001

Midterm Exam in Microeconomics

Saturday, July 28, 2001, 09.00h - 11.00h

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.

2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously. Please use **a pen** rather than a pencil so that your answers can be read without problems.

3. Good luck!

ID-Number:_____

Studienzentrum Gerzensee Doctoral Program in Economics
Microeconomics Midterm Exem

Klaus Schmidt and Jean-Charles Rochet

July, 2001

1. (20 points) A consumer's utility function is given by

$$u(x_1, x_2) = \beta_1 \ln(x_1 - \gamma_1) + \beta_2 \ln(x_2 - \gamma_2)$$

where $\beta_1, \beta_2 > 0$.

(a) Can we assume (without loss of generality) that $\beta_1 + \beta_2 = 1$?

(b) Compute the consumer's Marshallian demand functions and show that his expenditures $(p_i x_i)$ for goods 1 and 2 give rise to a linear expenditure system. Assuming that $\gamma_1, \gamma_2 \geq 0$, how can this expenditure system be interpreted?

(c) Under what assumptions on γ_1 and γ_2 are the preferences of the consumer homothetic?

2. (20 points) Patrick's utility function is $u(x_1, x_2) = x_1 \cdot x_2$, where good 1 is food and good 2 is housing. Patrick gets a monthly salary of \$3000. The price of good 1 and the price of good 2 are $p_1 = p_2 = 1$. Patrick's boss is thinking of sending him to another town where the price of food is the same, but the price of housing is 2.25. The boss offers no raise in pay. Patrick, who understands compensating and equivalent variation perfectly, complains bitterly. He says that although he doesn't mind moving for its own sake and the new town is just as pleasant as the old, having to move is as bad as a cut in pay of \$A. He also says he wouldn't mind moving if - when he moved - he got a raise of \$B. What are A and B equal to?

3. (20 points) Let $c(w_1, w_2, y)$ be the cost function of a firm, where w_1 and w_2 are the prices of input factors 1 and 2, and y is the quantity to be produced. Let $z_1(w_1, w_2, y)$ denote the conditional factor demand function of this firm.

(a) Show that

$$\frac{\partial c(w_1, w_2, y)}{\partial w_1} = z_1(w_1, w_2, y)$$

(b) Suppose that the firm can use two different production functions. Keeping input prices fixed, these production functions give rise to cost functions $c_1(y_1) = y_1^2/2$ and $c_2(y_2) = y_2$. Suppose that the firm is free how to allocate production between these two production functions, how does its cost function $c(y)$ look like?

4. (60 points) We consider an economy with three goods: labor (taken as a numeraire), and two consumption goods indexed $i = 1, 2$ whose prices are denoted p_1 and p_2. The first consumption good is produced by firm 1, according to the production function

$$y_1 = \sqrt{2l_1}$$

where y_1 is the quantity of good 1 produced with l_1 units of labor. Labor is thus the unique input used for the production of good 1. The second consumption good is produced by firm 2, according to the production function

$$y_2 = \sqrt{zl_2}$$

where y_2 is the quantity of good 2 produced with z units of good 1 and l_2 units of labor. Good 1 is thus used both as a consumption good and as an input for the production of good 2. Both firms are supposed to behave competitively.

(a) Compute the demand for labor, the supply of good 1 and the profit of firm 1 as a function of p_1.

(b) Show that the profit function of firm 2 has a maximum only when $p_1 = \frac{1}{4}p_2^2$. Show that in this case

$$\frac{l_2}{z} = \left(\frac{2p_1}{p_2}\right)^2$$

and that the maximum of the profit is zero. Can you explain why it is so?

There is a unique consumer with an inelastic supply of labor L and a utility function $U = x_1^a x_2^{1-a}$, with $0 < a < 1$. There are no initial endowments of the consumption good.

(c) Compute the demand functions of the consumer as a function of p_1, p_2 and his income I.

(d) Compute the competitive equilibrium of this economy.

(e) Show that the equilibrium prices of the two goods both increase when the parameter a increases, i.e. when the relative preference for good 1 increases. Can you explain why it is so?

(f) (optional) Compute the new competitive equilibrium prices when the production function of firm 1 is replaced by $y_1 = l_1$. Can you explain why they do not depend on L?