Studienzentrum Gerzensee Doctoral Program in Economics
Midterm Exam in Econometrics

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Write your Identification Number in the space provided below. (Do not give us your name – just your ID Number.)

Identification Number: ____

There are 120 points on this 120-minute exam. The number of possible points for each question is shown in parentheses preceding the question. Please answer all questions on the exam sheet. If you need additional space use the back of the exam sheet. Feel free to use your notes and any textbooks that you may find useful.

(20) (1) X and Y have joint density function $f_{X,Y}(x,y) = \frac{1}{8}(x+y)$ for $0 \leq x \leq 2$ and $0 \leq y \leq 2$, and $f_{X,Y}(x,y) = 0$ elsewhere.

(7) (a) What is the density of X?

(5) (b) Let $Z = 3 + X^2$. What is the probability density of Z?

(8) (c) Suppose $X = 1$. What is your best guess of the value of Y? (Explain what "best guess" means and provide a numerical answer.)

(30) 2. Let Y_i, i = 1, … , n denote an i.i.d. sample of Bernoulli random variables with $\Pr(Y_i = 1) = p$. I am interested in a parameter θ that is given by $\theta = e^p$.

(7) (a) Construct the likelihood function for θ.

(7) (c) Show that $\hat{\theta}_{MLE} = e^{\bar{Y}}$, where $\bar{Y}$ is the sample mean of the Y_i's.

(8) (d) Show that $\hat{\theta}_{MLE}$ is consistent

(8) (e) Show that $\sqrt{n}(\hat{\theta}_{MLE} - \theta) \xrightarrow{d} N(0,V)$, and derive an expression for *V*.

(10) (3) Y_i, $i = 1, \dots, 100$ are i.i.d. Poisson random variables with mean m. I am interested in the competing hypotheses H_o: $m = 4$ versus H_a: $m > 4$. I decide to reject the null hypothesis if $\bar{Y} > 4.2$. Use an appropriate large-sample approximation to compute the size of my test. (Hint, recall that the variance is equal to the mean for a Poisson random variable.)

Problem 4 (22 points)

Consider the model

$$y_i = z_i\delta + \varepsilon_i, \qquad \text{with} \qquad E\left[\varepsilon_i x_i\right] = 0$$

Assume that z_i is one–dimensional, and that x_i is two–dimensional with $x_i = \begin{pmatrix} z_i \\ \widetilde{x}_i \end{pmatrix}$ (so the explanatory variable is one of the instruments). Assume that assumptions (3.1)–(3.5) of Hayashi are satisfied and that

$$E\left[x_i x_i'\right] = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

and

$$E\left[\left.\varepsilon_i^2\right| x_i\right] = 1$$

(a) (5 points) Find the asymptotic distribution of the OLS estimator of δ.

(b) (5 points) Find the asymptotic distribution of the 2SLS estimator of δ that uses x_i as instrument.

(c) (5 points) Find the asymptotic distribution of the 2SLS estimator of δ that uses only $\widetilde{x}_i$ as instrument.

(d) (7 points) Discuss how the estimators in (a)–(c) compare in terms of the asymptotics.

Problem 5 (11 points)

An economist runs two regressions

$$y_i = \beta_0 + x_{1i}\beta_1 + x_{2i}\beta_2 + x_{3i}\beta_3 + \varepsilon_i \tag{1}$$

and

$$y_i = \beta_0 + x_{2i}\beta_2 + \nu_i \tag{2}$$

The results are summarized in the following table:

Equation 1.		
Variable	Coefficient	Std. Error
β_0	0.45	0.61
β_1	-0.62	0.30
β_2	5.62	0.82
β_3	0.36	0.39
R-squared	0.201	
Sum squared resid	14303	
Number of observations	200	
Equation 2.		
Variable	Coefficient	Std. Error
β_0	0.29	0.61
β_2	5.49	0.83
R-squared	0.182	
Sum squared resid	14658	
Number of observations	200	

Answer the following questions under the assumption that the usual regularity conditions for the finite sample properties of OLS are satisfied (assumptions 1.1-1.5 of Hayashi). Note that a 100% correct answer assumes that you have access to statistical tables. Do not worry about that. You will get full credit as long as you describe what you would look up in a table if you had one.

(a) (3 points) Consider Equation 1. Construct a 95% confidence interval for β_2

(b) (3 points) Test whether β_1 in Equation 1 equals 0 (against the alternative that it differs from 0). Test at a 5% level of significance.

(c) (5 points) Test the hypothesis that both β_1 and β_3 in Equation 1 equal 0 (against the alternative that it differs from 0). Test at a 5% level of significance.

Problem 6 (10 points)

Suppose that you have a random sample of (y_i, x_i) from

$$y_i = \beta_1 + x_i\beta_2 + x_i^2\beta_3 + \varepsilon_i$$

where x_i and ε_i are independent and $E[x_i] = E[x_i^3] = 0$. Answer the following under the assumption that all relevant moments exist.

Suppose that you regress y_i on a constant and x_i. Let the coefficient be denoted by b_1 and b_2, respectively.

(a) (4 points) What can you say about consistency of b_2 (as an estimator of β_2)?

(b) (4 points) What can you say about unbiasedness of b_2 (as an estimator of β_2)?

(c) (4 points) Find the joint asymptotic distribution of (b_1, b_2).

(d) (5 points) Suppose you are interested in testing $\beta_2 = 0$. Could you use the t–test based on b_2? What if you used the Eicker–Huber–White standard errors?