Macroeconomics Midterm Exam

Bob King and Sergio Rebelo
Swiss Program For Beginning Doctoral Students in Economics
Studienzentrum Gerzensee

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1. Short Question: Rational expectations in practice Suppose that an asset price p_t is determined according to

$$p_t = ¯E_t[p_{t+1} + d_{t+1}]$$

where ¯ = $\frac{1}{1+r}$ is a discount factor based on a constant real interest rate r and d_t is the dividend on the asset at date t.

(a) Assuming r > 0, derive the conclusion that the asset price is the present discounted value of future dividends.

(b) Further assume that dividends are governed by d_t = ½$d_{t ¡ 1}$ + e_t with e_t being a random variable with $E_t e_{t+j}$ = 0 for all t and for all j > 0. Assume that 0 < ½ · 1. Derive a formula which links p_t to d_t. Provide an economic interpretation of this formula..

2. Short Question: Rational expectations in theory Consider the following rational expectations model.

$$E_t \begin{bmatrix} y_{1,t+1} \\ y_{2,t+1} \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}$$

Under each of the following conditions, is there (1) a unique, stable RE equilibrium; (2) multiple stable RE equilibria; or (3) no stable RE equilibria?

(a) $0 < a < 1$, y_{1t} predetermined and $|b| > 1$, y_{2t} nonpredetermined.

(b) $0 < a < 1$, y_{1t} non predetermined and $|b| > 1$, y_{2t} predetermined.

(c) $0 < a < 1$, y_{1t} predetermined and $|b| < 1$, y_{2t} nonpredetermined.

3. **Short Question: Monopolistic Competition** Consider an economy where final output producers are perfectly competitive and use a continuum of intermediate goods, x_i, according to the following production function:

$$Y = \left(\int_0^1 x_i^{\alpha} di \right)^{1/\alpha}.$$

Each intermediate good is produced by a different monopolist who charges a price p_i for his good.

(a) Derive the first-order condition for the profit maximization of output producers.

(b) The cost of producing each intermediate good x_i in units of final output is given by $a + bx_i$, where a is the fixed cost and b the variable cost. Specify the profit maximization problem for the typical monopolistic producer and derive the optimal price charged by this producer.

4. **Short Question: Small Open Economy** Consider the following continuous-time planner's problem for a small open economy.

$$\begin{aligned} \max U &= \int_0^{\infty} e^{-\rho t} u(C_t) dt \\ \text{s.t.} \; \dot{a}_t &= r a_t - I_t - C_t + A K_t^{\alpha} \\ \dot{K}_t &= I_t - \delta K_t \\ K_0 &> 0 \text{ and } a_0 \text{ given.} \end{aligned}$$

Assume that $r = \rho$ and compute the steady state level of the capital stock K.

1. Long Question: Consumption over Time Consider a household which maximizes

$$E_t \sum_{j=0} {}^{-j} u(c_{t+j})$$

subject to a budget constraint

$$p(\pm_t)a_{t+1} + c_t = a_t + y(\pm_t)$$

in each period. In this expression, wealth at date t is a_t and income at date t is y_t. The household can buy future wealth a_{t+1} at price p_t: in the real world, this could involve buying a one period discount bond. The notation is designed to suggest that both income and the discount bond price are functions of a vector of exogenous state variables, $\pm_t$, that the household views as evolving according to a Markov process.

(a) Write down the Bellman equation for a dynamic programming analysis of the optimal consumption problem. What is the controlled state variable for the household?

(b) Find an e¢ciency condition that links together consumption at t and consumption at t+1. Discuss the economics behind this condition.

(c) Discuss the restrictions on this condition that are necessary to rationalize Hall's test of the permanent income theory of consumption.

(d) Loglinearize this condition to derive a link between consumption at the two dates and an additional variable. Discuss how this condition might be used to generalize Hall's test.

2. **Long Question: The Neoclassical Growth Model** Consider the following planner's problem for a version of the neoclassical growth model:

$$\begin{aligned} \max U &= \sum_{t=0}^{\infty} \beta^{t} \log(C_t) \\ \text{s.t. } K_{t+1} &= A K_t^{1-\alpha} N^{\alpha} - C_t \\ & K_0 \text{ given.} \end{aligned}$$

(a) What is the rate of depreciation in this economy?

(b) Derive the first-order conditions for the planner's problem.

(c) Specify the transversality condition for the planner's problem. What is the meaning of this condition?

(d) Under what condition will lifetime utility, U, be …nite in this economy?

(e) Compute the steady state level of the capital stock.

(f) What is the steady state real interest rate in this economy?

(g) Suppose that the is a permanent increase in the level of productivity, A. What is the impact of this shock on the steady state capital stock? And on the steady state real interest rate?

(h) Verify that the solution to the planner's problem has the form: $C_t = \mu A K_t^{1-\alpha} N^{\alpha}$ and compute μ.

(i) Linearize the first-order conditions for the planner's problem around the steady state.