Studienzentrum Gerzensee Doctoral Program in Economics
Midterm Econometrics Exam

Bo Honoré and Mark Watson

July, 2003

Instructions:

Write your Identification Number in the space provided below. (Don't give us your name, just your ID number.)

ID Number: ____

There are 120 points on this 120 minute exam. The number of possible points for each problem is shown in parentheses. If you need additional space use the back of the exam sheet. Feel free to use your notes and any textbooks that you may find useful.

(10) 1. Let $M(t)$ denote the mgf of X. Let $S(t) = \ln(M(t))$. Let $S'(0)$ denote the first derivative of $S(t)$ evaluated at $t = 0$ and $S''(0)$ denote the second derivative.

(5) (a) Show that $S'(0) = E(X)$

(5) (b) Show that $S''(0) = \text{var}(X)$

(7) 2. Suppose that the joint distribution of X and Y are two random variables with joint normal distribution. Further suppose $var(X) = var(Y)$. Let $U = X + Y$ and $V = X - Y$. Prove that U and V are independent.

(10) 3. Let $\bar{X}_1$ and $\bar{X}_2$ denote the sample means from two independent randomly drawn samples Bernoulli random variables. Both of the samples is size *n* and the Bernoulli population has P(*X* = 1) = 0.5. Use an appropriate approximation to determine how large *n* must be so that

$$P(|\bar{X}_1 - \bar{X}_2| < .01) = .95$$

(18) 4. Let Y_i , $i = 1, \ldots, n$ be i.i.d. $N(\mu, \sigma^2)$ random variables. Let $\theta = \mu^2$.

(4) (a) What is the MLE of θ?

(4) (b) Is $\hat{\theta}_{MLE}$ unbiased?

(5) (c) Show that $\sqrt{n}(\hat{\theta}_{MLE} - \theta_0) \xrightarrow{d} N(0, V)$ and derive an expression for *V*.

(5) (d) From a sample of size *n* = 100, I compute $\overline{Y} = 52$ and $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \overline{Y})^2 = 4$.

Construct a 95% confidence interval for θ (using an appropriate approximation if necessary.)

(15) 5. A coin is to be tossed 4 times and the outcomes are used to test the null hypothesis that $H_o : p = ½$ vs. $H_a: p > ½$, where p denotes the probability that a "tails" appears. ("Tails" is one surface of the coin, and "heads" is the other surface.)

I decide the reject the null hypothesis if "tails" appears in all four of the tosses.

(5) (a) What is the size of this test?

(5) (b) Sketch the power function of this test for values of $p>1/2$.

(5) (c) Is this the best test to use? Explain.

Problem 6 (36 points)

(Note that many of the questions in the problem can be answered independently of each other)

An economist runs the regression

$$y_i = \beta_0 + x_{1i}\beta_1 + x_{2i}\beta_2 + x_{3i}\beta_3 + \varepsilon_i \tag{1}$$

The results are summarized in the following table:

Equation 1.		
Variable	Coefficient	Std. Error
β_0	0.82	1.04
β_1	-1.16	5.74
β_2	7.63	3.70
β_3	-2.57	0.94
R-squared	0.102	
Sum squared resid	10297	
Number of observations	100	

Answer questions (a) through (d) under the assumption that the usual regularity conditions for the finite sample properties of OLS are satisfied (assumptions 1.1-1.5 of Hayashi). Note that a 100% correct answer assumes that you have access to statistical tables. Do not worry about that. You will get full credit as long as you describe what you would look up in a table if you had one.

(a) (3 points) Construct a 95% confidence interval for β_2 in Equation 1.

(b) (3 points) Test whether β_1 in Equation 1 equals 0 (against the alternative that at least one of them differs from 0). Test at a 5% level of significance.

(c) (5 points) Construct an estimate of the unconditional variance of y_i

Using the same data, the economist also runs the regression

$$y_i = \beta_0 + x_{2i}\beta_2 + \nu_i \tag{2}$$

Some of the results are summarized in the following table:

Equation 2.		
Variable	Coefficient	Std. Error
β_0	0.83	1.07
β_2	6.86	3.81
R-squared	?	
Sum squared resid	11095	
Number of observations	100	

(d) (5 points) Test the hypothesis that both β_1 and β_3 in Equation 1 equal 0 (against the alternative that it differs from 0). Test at a 5% level of significance.

(e) (15 points) The usual assumptions include (using the same numbering of the assumptions as Hayashi)

(1.3) $E\left[\varepsilon_i \middle| x_1, x_2, ..., x_n\right] = 0$

(1.4) $V\left[\varepsilon_i \middle| x_1, x_2, ..., x_n\right] = \sigma^2 I$

(1.5) $(\varepsilon_1, \varepsilon_2, ..., \varepsilon_n)'$ is normally distrubuted conditional on $(x_1, x_2, ..., x_n)$

How would your answers to (a), (b) and (d) change if

(i) (1.3) and (1.4) are maintained, but one is not willing to assume (1.5)?

(ii) (1.3) and (1.5) are maintained, but one is not willing to assume (1.4)?

(iii) (1.4) and (1.5) are maintained, but (1.3) is changed to $E\left[\varepsilon_i \middle| x_1, x_2, ..., x_n\right] = c \neq 0$

(f) (5 points) What is R^2 in equation 2?

Problem 7 (15 points)

Consider the model

$$y_i = z_i\delta + \varepsilon_i, \qquad \text{with} \qquad E[\varepsilon_i z_i] = 0$$

Also assume that there is a second variable, w_i, such that $E[w_i\varepsilon_i] = 0$ and $E[z_i w_i] \neq 0$ (both z_i and w_i are one–dimensional) Assume that assumptions (3.1)–(3.5) of Hayashi are satisfied with $x_i = \begin{pmatrix} z_i \\ w_i \end{pmatrix}$.

Find the joint asymptotic distribution of the OLS estimator and the two–stage least squares estimator that uses w_i (but not z_i) as an instrument.

Problem 8 (9 points)

Consider the linear regression model

$$y_t = x_t'\beta + \varepsilon_t.$$

Assume that$\{\varepsilon_t, x_t\}$ is stationary and ergodic with finite second moment.

Suppose that $\widehat{\beta}$is a consistent estimator of β (but it is not necessarily the OLS estimator). Is

$$\widehat{\rho} = \frac{(T-1)^{-1}\sum_{t=2}^{T}\left(y_t - x_t'\widehat{\beta}\right)\left(y_{t-1} - x_{t-1}'\widehat{\beta}\right)}{T^{-1}\sum_{t=1}^{T}\left(y_t - x_t'\widehat{\beta}\right)^2}$$

necessarily a consistent estimator of the correlation between ε_t and ε_{t-1}? Explain.