Macroeconomics Midterm Exam

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1. Short Question: Forecasting Investment (10 points)

Suppose that investment (i_t) is governed by

$$i_t = \theta(E_t y_{t+1} - y_t)$$

where y_t is current income and $E_t y_{t+1}$ is a rational expectation of next period's output. Suppose that output is given by

$$y_t = m y_{t-1} + e_t$$

where e_{t+j} has mean zero given information at date t for all $j > 0$.

(a) What is the rational forecast of $E_t y_{t+1}$? on $E_t y_{t+j}$?

(b) What is the effect of an $e_t > 0$ on:

i_t :

$E_t i_{t+1}$:

$E_t i_{t+2}$:

2. Short Question: Multiple Equilibria in Rational Expectations Models (10 points) Consider the following rational expectations model.

$$bE_t y_{t+1} = y_t - cx_t$$

with $c > 0$.

(a) Suppose that $0 < b < 1$ and y_t is not predetermined. What is the stable, forward-looking solution for y_t? What is the effect of x_t on y_t holding fixed expectations about future x's?

(b) Now suppose that $b > 1$ and that y_t is not predetermined. Determine the family of stable rational expectations. What is the effect of x_t on y_t? on y_{t+1}?

(c) Are the two examples above consistent or inconsistent with the following assertion: "The relationship between endogenous and exogenous variables is not much affected by whether there is a unique stable rational expectations equilibrium or there are multiple equilibria."

3. Short Question: Monopolistic Competition (10 points) Consider an economy where there is a continuum of measure one of final output producers who are perfectly competitive. Each final good producer uses a continuum of intermediate goods, x_i, according to the following production function:

$$Y = \left[\int_0^n x_i^{\alpha} di \right]^{1/\alpha}.$$

Each intermediate good is produced by a different monopolist who charges a price p_i for his good. There is a continuum of measure n of monopolists.

(a) Derive the first-order condition for the profit maximization of output producers.

(b) Producing each intermediate good x_i requires using $N_i = a + bx_i$ units of labor. The cost of each unit of labor is the real wage, w. Specify the profit maximization problem for the typical monopolistic producer and derive the optimal price charged by this producer.

(c) Suppose that there is free entry into the intermediate goods market, so that each intermediate good producer makes zero profit. Assume that there are N units of labor that are inelastically supplied. Compute the production of each monopolist, x_i, and the equilibrium number of intermediate good producers, n.

4. Short Question: Small Open Economy (10 points) Consider the following planner's problem for a small open economy.

$$\begin{aligned}
&\max U = \sum_{t=0}^{\infty} \beta^t u(C_t) \\
&\text{s.t. } a_{t+1} = (1+r)a_t - I_t - C_t + AK_t^{\alpha} \\
&K_{t+1} = I_t + (1-\delta)K_t \\
&K_0 > 0 \text{ and } a_0 \text{ given,} \\
&\lim_{t\to\infty} \frac{a_t}{(1+r)^{t+1}} = 0.
\end{aligned}$$

Compute the steady state level of the capital stock K. Explain why the steady state capital stock is independent of the discount factor, β.

1. Long Question: The Neoclassical Growth Model (40 points) Consider the following planner's problem for a version of the neoclassical growth model:

$$\begin{aligned} \max U &= \sum_{t=0}^{\infty} \beta^t \log(C_t - \frac{1}{\nu} N_t^{\nu}) \\ \text{s.t. } Y_t &= I_t + C_t \\ Y_t &= A K_t^{1-\alpha} N_t^{\alpha} \\ K_{t+1} &= I_t + (1-\delta) K_t \\ & K_0 \text{ given.} \end{aligned}$$

where $\nu > 1$.

(a) Derive the first-order conditions for the planner's problem.

(b) Compute the steady state level of the capital stock.

(c) What are the steady state real interest rate and real wage rate for this economy?

(d) Linearize the first-order conditions for the planner's problem around the steady state.

(e) Suppose that there is a permanent increase in the level of productivity, A. What is the impact of this shock on the steady state real interest rate and real wage rate? And on the steady state labor supply?

(f) Modify the production function so as to introduce exogenous technical progress:

$$\begin{aligned} Y_t &= AK_t^{1-\alpha}(N_t X_t)^{\alpha}, \\ X_{t+1} &= \gamma X_t, \\ X_0 &> 0, \text{ given.} \end{aligned}$$

Does the resulting economy have a steady state growth path?

2. Long Question: Optimal Consumption with Occasionally Adjusting Consumers (40 points) There has been much recent interest in macroeconomic models that depart from the neoclassical benchmark in various specific ways. Because these models are motivated by observations about individual behavior, the research is sometimes called "behavioral macroeconomics" and it is sometimes called research on "macroeconomics and individual decision-making." This research develops the implications of well-specified alternatives to the neoclassical benchmark.

Perhaps one of the most famous neoclassical benchmarks is Hall's (1978) theory of consumption, in which consumers update their consumption paths in response to information about future income each period. While this theory has had some success at the macro level, it has been criticized by some economists as being inconsistent with how indvidual households actually process information.

A. The Neoclassical Benchmark

Consider a household which maximizes

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j})$$

subject to a budget constraint

$$\frac{1}{1+r} a_{t+1} + c_t = a_t + y(\delta_t)$$

in each period. In this expression, wealth at date t is a_t and income at date t is y_t. The household can buy future wealth a_{t+1} at price $\frac{1}{1+r}$. Income is a function of a vector of exogenous state variables, δ_t, that the household views as evolving according to a Markov process.

(i) Write down the Bellman equation for a dynamic programming analysis of the optimal consumption problem. What is the controlled state variable for the household?

(ii) Find an efficiency condition that links together consumption at t and consumption at t+1. Discuss the economics behind this condition.

(iii) Suppose that the utility function is of the form $u(c_t) = \gamma + \phi c_t - \frac{\theta}{2} c_t^2$ and that$\beta(1 + r) = 1$. Show that consumption is a random walk, i.e., that it evolves according to

$$c_{t+1} = c_t + e_{t+1}$$

where e_{t+1} is unpredictable given δ_t.

B. A Behavioral Alternative

Suppose that the population is divided into four equally sized groups of individuals. Individuals of type 1 adjust their consumption plans only in the first quarter of each year; individuals of type 2 adjust in the second quarter; and so forth. Call the date t consumption of each type of individual c_{jt}, Suppose that the time index denotes quarters of a year, t=1,2,3,4,5,

(iv) Why would this modification lead to a consumption behavior at the individual level which takes the form

$$\begin{aligned} c_{1t} &= c_{1,t-4} + \eta_{1t} \text{ if } t = 1, 5, 9, 13, ... \\ c_{1t} &= c_{1,t-4} \text{ if } t \neq 1, 5, 9, 13, ... \end{aligned}$$

for individuals of type 1, where η_{1t} is related to the revision in "permanent income" based on information that arrives between the date of the individuals last adjustment (t-4) and the current period (t).

(v) Assuming that aggregate consumption in any quarter is given by

$$c_t = \frac{1}{4}[c_{1t} + c_{2t} + c_{3t} + c_{4t}]$$

explain why it is plausible that this aggregate evolves according to

$$\begin{aligned}
c_t &= c_{t-1} + \frac{1}{4}[\eta_{1t}] \text{ if if } t = 1, 5, 9, 13, ... \\
c_t &= c_{t-1} + \frac{1}{4}[\eta_{2t}] \text{ if if } t = 2, 6, 10, 14, ... \\
c_t &= c_{t-1} + \frac{1}{4}[\eta_{3t}] \text{ if if } t = 3, 7, 11, 15, ... \\
c_t &= c_{t-1} + \frac{1}{4}[\eta_{4t}] \text{ if if } t = 4, 8, 12, 16, ...
\end{aligned}$$

(vi) Why might one expect that the η_t could be predicted by an econometrician using aggregate data from t-1, but not using aggregate data from t-4?

If this were true, how could the macroeconomic implications of the behavioral model be distinguished from those of the Hall model? How could the behavioral model be tested in a manner similar to the Hall-test?