1. (10 points) Show that if the demand function $x(p, w)$ is generated by a rational preference relation, then it must satisfy the weak axiom.

2. (20 points) John's utility function is $U(x, y) = \min\{x, y\}$. John has \$150 and the price of x and the price of y are both 1. John's boss is thinking of sending him to another town where the price of x is 1 and the price of y is 2. The boss offers no raise in pay. John, who understands compensating and equivalent variation perfectly, complains bitterly. He says that although he doesn't mind moving for its own sake and the new town is just as pleasant as the old, having to move is as bad as a cut in pay of \$A. He also says he wouldn't mind moving if - when he moved - he got a raise of \$B.

   (a) What are A and B equal to? Interpret them.

(b) Suppose that John gets a raise of $B. Can he still a¤ord his old consumption bundle? Explain why or why not. Would your answer still hold if John's utility function was $U(x, y) = x ¢ y$?

3. (30 points) Consider an agent who has initial wealth W. He can invest an amount A in a risky asset, yielding a stochastic return rA, where r is distributed according to the cumulative distribution function $F(r)$. The remainder of his wealth is held as money earning no interest. Assume that $\int_r r\,dF(r) > 1$, so there is an interior solution for the optimal level of A.

(a) Suppose his vNM utility function is

$$u(x) = -\alpha x^2 + \beta x, \qquad \alpha > 0, \beta > 0,$$

where x denotes his final wealth. Determine the coefficients of absolute and relative risk aversion. Show that the optimal value of A decreases with W.

(b) Suppose his vNM utility function is $v(x) = -e^{-cx}$. Determine the coefficients of absolute and relative risk aversion. Show that the optimal value of A is independent of W and decreases with c. When $\tilde{r}$ is distributed normally with mean μ and variance σ^2, show that the optimal value of A is increasing in μ. [Remainder: The density function of a normally distributed random variable is given by $f(r) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{r-\mu}{\sigma}\right)^2}$].

4. (30 points) Consider a …rm producing a quantity Q of some consumption good with capital K and labor L. The production function is

$$Q = KL.$$

The price of labor is denoted w, the price of capital is denoted r.

(a) What combination of labor and capital minimizes the cost of producing Q units of output?

(b) Deduce the cost function $C(w, r, Q)$.

(c) What can you say about returns to scale?

5. (30 points) Consider an economy with 2 goods: a numeraire and a consumption good. There are I consumers with identical utility functions:

$$u_i = 2\sqrt{x_i} + m_i.$$

There are J firms with identical cost functions $C_j(q_j) = \frac{\alpha}{2}q_j^2$.

The initial endowment of numeraire is M (shared equally between the consumers).

(a) Determine the aggregate supply function of the economy. The price of the good is denoted p.

(b) Determine the aggregate demand function of the economy.

(c) Find the general equilibrium price and quantities. How do they depend of $\alpha, I, J.$ Are these properties intuitive?