STUDY CENTER
GERZENSEE

Swiss Program for Beginning Doctoral Students in Economics 2004

Midterm Exam in Econometrics

Saturday, July 31, 2004, 14.00h - 16.00h

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.

2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.

3. Please use **a pen** rather than a pencil so that your answers can be read without problems.

4. Please **write legibly**. Remember that your exams will be photocopied for grading.

5. Answers should be **concise and precise**! The space provided should be sufficient to answer each question.

6. Good luck!

ID-Number:____

Problem 1 (20 points)

Suppose $f_{X,Y}(x,y) = ¼$ for $1 < x < 3$ and $1 < y < 3$, and $f_{X,Y}(x,y) = 0$ elsewhere.

(a) (7 points) Derive the marginal densities of X and Y.

(b) (4 points) Find $\mathbf{E}(Y \mid X = 2.3)$

(c) (4 points) Find $P[\,(X < 2.5) \cup (Y < 1.5)\,]$

(d) (5 points) Let $W = X + Y$. Derive the moment generating function of W.

Problem 2 (20 points)

X_i are *i.i.d.* $N(0,\ s^2)$. Let $s^2 = \dfrac{1}{n}\sum_{i=1}^{n} X_i^2$. I am interested in the competing hypotheses H_o: $s^2 = 1$ versus H_a: $s^2 > 1$. Suppose $n = 15$ and I decide to reject the null hypothesis if $s^2 > 5/3$.

(a) (5 points) What is the size of the test?

(b) (5 points) Suppose that the true value of s^2 is 1.5. What is the power of the test? (If you can't provide a numerical answer, just tell me how I can calculate the power.)

(c) (10 points) Is this the most powerful test? (Should I use another test instead?) Explain.

Problem 3 (10 points)

Suppose X is distributed with CDF $F(x)$ and density $f(x)$. Show that $\mathrm{E}(X) = \int_0^1 F^{-1}(t)dt$.

Problem 4 (5 points)

Consider the stationary AR(1) model $y_t = a + fy_{t-1} + e_t$. Show that the mean of y_t = $a/(1-f)$.

Problem 5 (5 points)

Consider the AR(2) model $(1-f_1\text{L}-f_2\text{L}^2)\ y_t = e_t$. Show that the roots of the AR polynomial are the reciprocals on the eigenvalues of the companion matrix.

Problem 6 (14 points)

Give the values for ?1–?7 in the following regression output. It is not necessary to provide any explanation.

Regression with robust standard errors

Number of obs	=	1000
F(2, 997)	=	171.57
Prob > F	=	0.0000
R-squared	=	0.2400
Root MSE	=	.36686

y	Coef.	Robust Std. Err.	t	[95% Conf.	Interval]
x1	-.4259559	?1	?2	-.8429015	?3
x2	1.119052	?4	?5	?6	1.338567
_cons	.8779869	.0546913	?7	.7706636	.9853101

?1 = 0.213

?2 = − 2.002   x1

?3 = −0.009

?4 = 0.112

?5 = 9.992   x2

?6 = 0.900

?7 = 16.0535   -const

Problem 7 (10 points)

Somebody regresses a variable y on a variable x (and a constant) and finds an R^2 of 0.05. If the sample size is 1000, do you conclude that y is unrelated to x?

$$t^2 = F = \frac{R^2}{(1-R^2)/(n-k)} \approx 52.$$

or $|t| > 7$ (very significant)

Problem 8 (20 points)

Suppose that you estimate an earnings equation using data from two different regions of the country. The results are

REGION 1:

$$\ln(earnings) = 6 + 0.11 \cdot \ln(education) - 0.03 \cdot female + \varepsilon$$

REGION 2:

$$\ln(earnings) = 6.5 + 0.08 \cdot \ln(education) - 0.02 \cdot female + \varepsilon$$

Here "education" is measured as the number of years of schooling, and "female" is a dummy variable that takes the value 1 for females and 0 for males. ε is an error–term.

(a) **(5 points)** For each of the two regions, what is the estimated effect on earnings of a 1 year increase in education for someone with 10 years of education (everything else being equal).

Reg 1 earnings will increase by approx $10 \times 0.11\%$ $= 1.1\%$

Reg 2 earning will increase by approx $10 \times 0.08\%$ $= 0.8\%$

[β_2 is the elasticity]

(b) **(5 points)** For each of the two regions, what is the estimated difference in earnings between males and females (everything else being equal).

Reg 1 3% lower for females

Reg 2 2% lower for females.

[β_3 is semi elasticity)

(c) **(5 points)** Explain in detail how you would test that the coefficients on log–education and on the female–dummy are the same in region 1 as they are in region 2 (while allowing the intercepts to differ). Please note that you are not asked to actually perform the test.

[redacted]. We would need to rerun a regression with the data pooled across regions of ln(earnings) on ln(education), female, a dummy variable for region, and interaction terms between region*ln(education) and region*female. To test the null hypothesis that the coefficients on ln(education) and female are the same across regions, we would run an F-test of the coefficients of the interaction terms.

(d) **(5 points)** Suppose the standard error on the coefficient on log–education in region 1 is 0.03. Construct a 95% confidence interval for the difference in expected log earnings between two females in region 1 having 9 and 12 years of education, respectively.

The difference in log- earnings is

$[\log 12 - \log 9] \cdot \beta_2$

So the CI is

$\{(\log 12 - \log 9)(0.11 - 1.96 \cdot 0.03),$
$(\log 12 - \log 9)(0.11 + 1.96 \cdot 0.03)\}$

or

$(0.015, 0.049)$

Problem 9 (16 points)

Consider the model

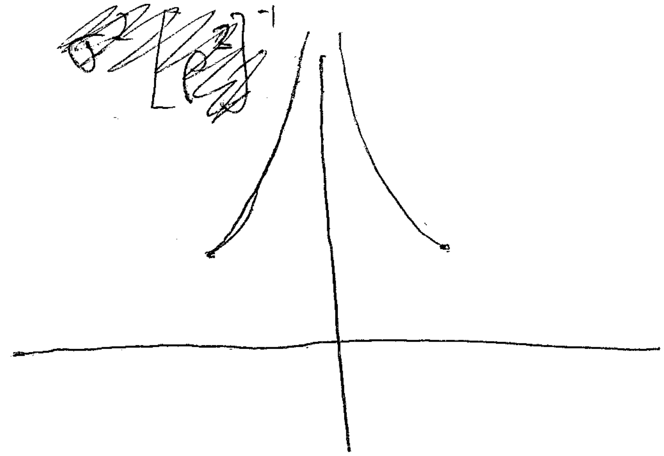
$$y_i = z_i\delta + \varepsilon_i, \qquad \text{with} \qquad E[\varepsilon_i x_i] = 0$$

Assume that both z_i and x_i are one–dimensional with $E[x_i] = E[z_i] = 0$, $V[x_i] = V[z_i] = 1$ and that the correlation between x_i and z_i is ρ. Finally assume that $E[\varepsilon_i^2 | x_i] = 1$ and that the regularity conditions in Hayashi (Assumptions 3.1–3.5) are satisfied.

(a) **(8 points)** Consider the 2SLS estimator of δ that uses x_i as instrument. Plot the variance of its asymptotic distribution as a function of ρ.

From Hayashi 3.8.4:

$$\text{Avar}(\hat{\delta}_{2SLS}) = \sigma^2(\Sigma_{xz}\Sigma_{xx}^{-1}\Sigma_{xz})^{-1} = 1[\rho \cdot 1 \cdot \rho]^{-1} = \frac{1}{\rho^2}$$



(b) **(8 points)** Now suppose that $E[\varepsilon_i x_i] = \kappa \neq 0$. Find the probability limit of the 2SLS estimator of δ that uses x_i as instrument (under the assumption that the remaining conditions are satisfied).

$$\hat{\delta}_{2SLS} = \left[S_{xz}' S_{xx}^{-1} S_{xz}\right]^{-1} S_{xz}' S_{xx}^{-1} s_{xy} =$$

$$\delta + \left[S_{xz}' S_{xx}^{-1} S_{xz}\right]^{-1} S_{xz}' S_{xx}^{-1} s_{x\varepsilon}$$

$$\xrightarrow{p} \delta + (\rho\, 1^{-1} \rho)^{-1} \rho \cdot 1^{-1} \cdot \kappa = \frac{\kappa}{\rho}$$