STUDY CENTER
**GERZENSEE**

# Swiss Program for Beginning Doctoral Students in Economics 2004

## Midterm Exam in Macroeconomics

## Friday, July 30, 2004, 14.00h – 16.00h

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.

2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously

3. Please use **a pen** rather than a pencil so that your answers can be read without problems.

4. Please **write legibly**. Remember that your exams will be photocopied for grading.

5. Answers should be **concise and precise**! The space provided should be sufficient to answer each question.

6. Good luck!

ID-Number:____________________________________________

**Question 1 [25 points]** *Labor Supply.*

Suppose that an individual has a utility function

$$u(c_t, n_t) = \log(c_t) - \frac{\chi}{1+\gamma} n_t^{1+\gamma}$$

and a budget constraint of the form

$$c = w(1-\tau)n + T$$

In these expressions, $c$ is consumption; $n$ is work effort; $w$ is the wage rate; $\tau$ is the tax rate on labor income; and $T$ is the level of transfer payments.

**(a)** Supposing initially that $T = 0$, compute the response of consumption and labor supply to a change in the tax rate and discuss the form that it takes.

**(b)** Now suppose that tax revenues are rebated lump sum to individuals, but do not depend on their individual level of labor supply. Instead, transfers depend on the level of labor supply by an average individual, with all agents having the same preferences and facing the same wage rates and tax rates. Hence, $T = \tau w \overline{n}$, where $\overline{n}$ is the average level of labor supply. Compute the response of consumption and labor supply to a change in the tax rate and discuss the form that it takes.

**Question 2 [25 points]** *Linear Rational Expectations Models.*

Consider the following two equation linear rational expectations model with $k_t$ predetermined and $i_t$ not predetermined.

$$\begin{aligned} k_{t+1} &= \mu k_t + \gamma i_t \\ i_t &= \beta E_t i_{t+1} + x_t \end{aligned}$$

**(a)** Determine the restrictions on the parameter values under which there is a unique stable solution.

**(b)** Provide an example of parameter values for which there are multiple stable RE equilibria and show the form of these equilibria.

**Question 3 [50 points]** *Dynamic Programming and Investment Demand.*

Suppose that a firm chooses investment ($i_t$) to maximize its market value. Investment is valuable because it changes the capital stock $k_t$ which is the firm's sole factor of production. There is time-varying productivity ($a_t$) and a time-varying price of investment ($p_t$). The firm's objective is to maximize

$$V = E_t \sum_{t=0}^{\infty} \beta^t [a_t k_t - p_t i_t]$$

subject to

$$k_{t+1} = h(\frac{i_t}{k_t})k_t$$

where $h(\frac{i_t}{k_t})$ is an increasing and concave function. This function will be assumed to have the properties that $h(0) = 1-\delta$ and that $h(\delta) = 1$ for some $\delta$ that is between zero and one. Finally, productivity and the investment good price are assumed to be functions of an exogenous state vector $\varsigma_t$, which evolves stochastically and is a Markovian.

**(a) (10 points)** Formulate a dynamic program for this firm, specifying the Bellman equation and indicating the firm's control and state variables.

**(b) (10 points)** Find one or more first order conditions for the firm's optimal investment. Provide an economic interpretation of each condition. Use the envelope theorem to determine the derivative of the value function.

**(c) (10 points)** Assuming that $a$ and $p$ are constant through time, find a steady state of a certainty version of this model. Find a transformation of the model's variables, such that these transformed variables are constant in steady state.

**(d) (20 points)** Now assume that $a$ and $p$ are stationary random variables. Using the results above, particularly (c), show how to generate an approximate loglinear rational expectations model from the FOCs and calculate the loglinear approximation to at least two of the equations (In your answer, you may find it convenient to write the model equations in terms $z_t = i_t/k_t$). If you were to solve this model, how many stable eigenvalues must there be if there is to be a unique solution? Would you expect this model to have a unit root? If so, would you treat it as stable or unstable?

**Question 4 [25 points]** *Growth Model.*

Consider the following growth model where agents can use units of output to invest in both physical ($K_t$) and human ($H_t$) capital:

$$\begin{aligned} U &= \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1-\sigma} \\ Y_t &= A K_t^{1-a} (N H_t)^{\alpha} \\ Y_t &= C_t + I_t^K + I_t^H \\ K_{t+1} &= I_t^K + (1-\delta) K_t \\ H_{t+1} &= I_t^H + (1-\delta) H_t \end{aligned}$$

**(a)** Compute the steady state growth rate for this economy.

**(b)** Suppose that the initial conditions $K_0$ and $H_0$ are not consistent with steady state growth. How long does it take for the economy to converge to the steady state growth path?

**Question 5 [25 points]** *The Solow Model with Investment-Specific Technical Progress.*

Consider the following version of the Solow model in which technological progress is investment specific and investment is a constant fraction of output.

$$
\begin{aligned}
U &= \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \\
Y_t &= C_t + I_t \\
Y_t &= K_t^{\alpha} \\
I_t &= sY_t \\
K_{t+1} &= (1-\delta)K_t + I_t q_t \\
q_t &= \gamma^t, \gamma > 1
\end{aligned}
$$

Compute the steady state growth rate of output, capital, and consumption.

**Question 6 [50 points]**

Consider the following problem for a representative consumer who lives in a small open economy that can borrow and lend at an interest rate $r$.

$$\max U = \sum_{t=0}^{\infty} \beta^t \log(C_t)$$

subject to:

$$\begin{aligned} a_{t+1} &= (1+r)a_t + Y - C_t(1+\tau) \\ \lim_{t\to\infty} \frac{a_t}{(1+r)^t} &= 0 \end{aligned}$$

where $Y$ is a constant, exogenous level of output and $\tau$ is a tax rate on consumption. The government maintains balance uses its tax revenue to finance government spending $G$:

$$\tau C_t = G_t$$

Assume that:

$$\beta = \frac{1}{1+r}$$

**(a)** Characterize the equilibrium path for total consumption (private plus public), $C_t + G_t$ in this economy.

**(b)** Characterize the behavior of the current account in the economy studied in (a).

**(c)** Suppose now that the tax rate on consumption follows the following time-varying path:

$$\tau_t = \begin{cases} \tau^* & \text{for } t \leq T \\ \tau^{**} & \text{for } t \geq T+1 \end{cases}$$

where $\tau^* < \tau^{**}$. The flow budget constraint for the household is now:

$$a_{t+1} = (1+r)a_t + Y - C_t(1+\tau_t)$$

The government adjusts government expenditures so that they coincide in every period with the tax revenue:

$$\tau C_t = G_t.$$

Show that the path for private consumption is as follows:

$$C_t = \begin{cases} C^* & \text{for } t \leq T \\ C^{**} & \text{for } t \geq T+1 \end{cases}$$

where:

$$C^*(1+\tau^*) = C^{**}(1+\tau^{**}).$$

Compute the value of $C^*$.

**(d)** Characterize the behavior of the current account in the economy studied in (c).