STUDY CENTER
GERZENSEE

Swiss Program for Beginning Doctoral Students in Economics 2004

Midterm Exam in Microeconomics

Saturday, July 31, 2004, 09.00h – 11.00h

1 You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.

2 Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.

3 Please use **a pen** rather than a pencil so that your answers can be read without problems.

4 Please **write legibly** Remember that your exams will be photocopied for grading.

5 Answers should be **concise and precise**! The space provided should be sufficient to answer each question.

6 Good luck!

ID-Number:____

Question 1

Consider an exchange economy with 2 consumers ($i = 1, 2$) and 2 goods ($l = 1, 2$). Initial endowments are $w_1 = (1, 2)$ for consumer 1 and $w_2 = (2, 2)$ for consumer 2. Their utility functions are:

$$u_1 = x_{11} \cdot x_{21}^2 \qquad \text{and} \qquad u_2 = x_{12} \cdot x_{22}$$

(a) Characterize the Pareto optima of this economy. Represent them in the Edgeworth box.

(b) Compute the particular Pareto optimum (denoted S) that maximizes the welfare function $W = u_1 u_2$ over all feasible allocations.

(c) Compute the competitive equilibrium of this economy.

(d) Find the system of transfers needed to implement S (the Pareto optimum that maximizes $W = u_1 u_2$) as a competitive equilibrium with transfers.

Question 2

Consider a production economy with one consumer, 2 goods ($l = 1, 2$) and 2 firms ($j = 1, 2$). Each firm transforms good 1 into good 2. The production functions are $y_{21} = \sqrt{-y_{11}}$ for firm 1 and $y_{22} = 2\sqrt{-y_{12}}$ for firm 2. (We adopt the classical convention that inputs are counted negatively). The utility function of the consumer is $u = x_1 x_2$ and his initial endowment is $w = (2, 0)$.

(a) Compute the competitive equilibrium of this economy.

(b) The two firms merge, and form a conglomerate. Compute the production function of the conglomerate.

(c) Compute the new competitive equilibrium of the economy, with the conglomerate as a unique firm.

(d) What do you notice? Explain.

Question 3 (15 points)

Consider a consumer who consumes only two goods and satisfies Walras' Law. When prices are $(4, 8)$ he demands $(15, 30)$. When prices are $(12, 6)$, he demands $(30, y)$. Nothing else of significance has changed between the two situations.

(a) Suppose that $y = 15$. Do these consumption plans satisfy the weak axiom of revealed preference.

(b) For which range of y do these consumption plans violate the weak axiom?

Question 4 (20 points)

Let x be a vector of inputs and w the corresponding vector of input prices. Suppose that the production function $f(x)$ has constant returns to scale.

(a) Show that the corresponding cost function can then be written as $c(w, y) = y \cdot c(w)$, where $c(w) = c(w, 1)$.

(b) Furthermore, show that the conditional input demand function $x(w, y)$ can be written as $x(w, y) = y \cdot x(w)$.

(c) Suppose now in addition that each factor x_i is paid its value of marginal product, $w_i = p\frac{\partial f(x)}{\partial x_i}$. Show that profits must be zero.

Question 5 (25 points)

Suppose that an agent's utility function has constant relative risk aversion with $r_R(w) = 1$.

(a) Show that this is the case if and only if the agent's utility function is a positive affine transformation of $u(w) = \ln w$.

(b) Assume that this agent faces a two-period portfolio allocation problem. In period $t \in \{0, 1\}$, his wealth w_t is to be divided between a safe asset with return R and a risky asset with return x (these are gross returns per unit of investment). The initial wealth at period 0 is w_0. Wealth at period $t = 1, 2$ depends on the portfolio α_{t-1} chosen at period $t - 1$ and on the return x_t realized at period t, according to

$$w_t = [(1 - \alpha_{t-1})R + \alpha_{t-1}x_t]w_{t-1} \ .$$

The objective of this individual is to maximize the expected utility of terminal wealth w_2. Assume that x_1 and x_2 are independently and identically distributed. Prove that the individual optimally sets $\alpha_0 = \alpha_1$.