1. Short Question: Rational Expectations Models (12 points, equal weights for each part) Suppose that the short-term interest rate is governed by

$$R_t = \rho R_{t-1} + e_t$$

where $E_t e_{t+1} = 0$ and $|\rho| < 1$. Suppose that the long-term interest rate is governed by

$$R_t^L = \frac{1}{2}[R_t + E_t R_{t+1}]$$

(a) The rational expectations solution for the long-term interest rate takes the form

$$R_t^L = \pi R_t$$

What is the value of π?

(b-1) Suppose that the vector of endogenous variables is $Y_t = [R_t^L \; R_t \; R_{t-1}]'$ and that this two equation model is written in the form $AE_tY_{t+1} = BY_t + CX_t$, where $X_t = e_t$. What are the matrices A,B,C?

(b-2) What are the roots of $|Az - B| = 0$? Given that there is one predetermined variable (R_{t-1}), is the model uniquely solvable?

2. Short Question: Consumption choice (12 points, equal weights for each part) Consider the following basic intertemporal model of consumption choice, in which a household maximizes the objective function

$$U = u(c_0) + \beta u(c_1)$$

with $u_c(c) > 0$ and $u_{cc} < 0$.

$$c_0 + \frac{1}{1+r}c_1 = y_0 + \frac{1}{1+r}y_1$$

(a) Show the following: If $\beta(1+r) = 1$, then $c_0 = c_1$.

(b) Show the following: If $\beta(1+r) = 1$ and income rises in both periods by the same amount dy, then

$$\frac{dc_0}{dy} = 1.$$

(c) If $\beta < 1$, do these properties carry over to an intertemporal model with

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t)$$

and

$$\sum_{t=0}^{\infty} \beta^t [y_t - c_t] = 0 \quad ?$$

3. Longer Question: Dynamic programming (36 points, equal weights for each part) Consider an economy in which the representative individual has preferences

$$\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} c_t^{1-\sigma}$$

and his capital accumulates according to

$$k_{t+1} = Ak_t - c_t$$

(a) Write the Bellman equation for optimal capital accumulation.

(b) Find the FOC(s) that must be satisfied for an optimal consumption/capital plan.

(c) Use the envelope theorem to determine $v_k(k)$.

(d) Show that a consumption policy of the form

$$c_t = Bk_t$$

is consistent with your results in (b) and (c). Determine the value of the coefficient B.

(e) What economic considerations govern the growth rate of consumption in this economy? How is the growth rate of capital related to the growth rate of consumption?

(f) How is this dynamic program related to material covered in weeks 1 and 2 of the course?

(g) Derive a candidate value function under the policy derived in part (d), showing that it takes the form

$$v(k) = w * k^{1-\sigma}$$

and determine the value of w.

4. Short Question: Growth Model (16 points) Consider the following growth model that features an externality. Output of an individual firm (y_t) depends on the capital (k_t) and labor (l) employed by this firm, as well as on the total level of production in the economy (Y_t):

$$y_t = Ak_t^{1-\alpha} l^\alpha Y_t^\gamma,$$

where $0 < \gamma < 1$. Each firm in the economy is small so it takes aggregate output, Y_t, as given. There are n firms in the economy, so:

$$\begin{aligned} Y_t &= ny_t, \\ K_t &= nk_t, \\ L &= nl, \end{aligned}$$

where K_t is the aggregate capital in the economy. There is no population growth, so the aggregate labor supply, L, is constant over time.

Total output in the economy can be used for consumption (C_t) or investment (I_t).

$$Y_t = C_t + I_t.$$

The law of motion for the aggregate capital stock is given by:

$$K_{t+1} = I_t + (1 - \delta)K_t.$$

Suppose that agents save at a constant rate, s:

$$I_t = sY_t.$$

(a) Characterize the values of α and γ such that the economy grows at a constant rate.

(b) Suppose that you are the planner for this economy. What is the optimal number of firms, n?

5. Short Question: A Small Open Economy (16 points) Consider the following problem for a representative consumer who lives in a small open economy that can borrow and lend at a constant real interest rate r.

$$\max U = \sum_{t=0}^{\infty} \beta^t \log(C_t)$$

subject to:

$$a_{t+1} = (1+r)a_t + Y_t - C_t$$
$$\lim_{t\to\infty} \frac{a_{t+1}}{(1+r)^t} = 0$$

The variable Y_t represents a constant, exogenous level of output. Agents in the economy expect Y_t to remain constant at a level Y. Assume that $\beta = 1/(1+r)$.

Discuss the reaction of consumption and the current account to the following two unanticipated events that occur at time zero.

(a) The economy learns that output will increase permanently from Y to $Y + \delta$, with $\delta > 0$.

(b) The economy learns that output will increase temporarily, following the path:

$$\tau_t = \begin{cases} Y + \delta & \text{for } t = 0, \\ Y & \text{for } t > 0 \end{cases}$$

with $\delta > 0$.

6. Longer Question: Fiscal Policy in the Neoclassical Growth Model (28 points) Consider the following version of the neoclassical growth model where τ_t represents lump sum taxes paid by the representative agent to the government. There is no population growth, so the number of agents in the economy remains constant over time.

$$\begin{aligned} U &= \sum_{t=0}^{\infty} \beta^t \frac{(C_t - \psi N_t^{\theta})^{1-\sigma} - 1}{1 - \sigma} \\ Y_t &= C_t + I_t + \tau_t \\ Y_t &= A K_t^{1-\alpha} N_t^{\alpha} \\ K_{t+1} &= I_t + (1 - \delta) K_t \end{aligned}$$

where $0 < \beta < 1$, $\theta > 1$, $\psi > 0$, $0 < \alpha < 1$, and $\sigma > 0$.

The government runs a balanced budget,

$$\tau_t = G_t,$$

where G_t represents the level of government spending.

(a) Assume that $G_t = G$ is constant. Compute the steady state levels of output, capital, and consumption.

(b) Suppose that the economy is in the steady state and learns that there will be an increase in G in the future. What is the immediate impact today of this news on the supply of labor and the level of output?

(c) Suppose that life-time utility takes the form:

$$U = \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \theta \log(1 - N_t) \right]$$

Assume, in addition that $\delta = 1$ and that taxes and government spending are proportional to output:

$$\tau_t = G_t = \gamma Y_t.$$

Show that the solution to the model takes the form

$$C_t = \mu Y_t$$

and compute the value of μ.