

Question 1 (20 points) Consider a utility function of the form $u(x_1, x_2) = (x_1 - \gamma_1)^{\alpha_1} \cdot (x_2 - \gamma_2)^{\alpha_2}$ which is defined for $x_l \geq \gamma_l$. Ignoring possible corner solutions, show that this utility function gives rise to a linear expenditure system:

$$\begin{aligned} p_1 x_1 &= p_1 \gamma_1 + \tilde{\alpha}_1 [w - p_1 \gamma_1 - p_2 \gamma_2] \\ p_2 x_2 &= p_2 \gamma_2 + \tilde{\alpha}_2 [w - p_1 \gamma_1 - p_2 \gamma_2] \end{aligned}$$

with $\tilde{\alpha}_1 + \tilde{\alpha}_2 = 1$. How would you interpret these utility and expenditure functions?

Question 2 (20 points) Let there be two goods with Walrasian demand for the first good given by $x_1 = \frac{1}{p}$, where p denotes the price of good 1 relative to price of good 2. Let income divided by price of good 2 be denoted by w .

- (a) What is the Walrasian demand for good 2?

(b) Find the indirect utility function $v(p, w)$.

(c) Find the direct utility function $u(x_1, x_2)$.

Question 3 (20 points) Let $c(w_1, w_2, y)$ be the cost function of a firm, where w_1 and w_2 are the prices of input factors 1 and 2, and y is the quantity to be produced. Let $z_1(w_1, w_2, y)$ denote the conditional factor demand function of this firm.

(a) Show that $\frac{\partial c(w_1, w_2, y)}{\partial w_1} = z_1(w_1, w_2, y)$.

(b) Suppose that the firm can use two different production functions. Keeping input prices fixed, these production functions give rise to cost functions $c_1(y_1) = y_1^2/2$ and $c_2(y_2) = y_2$. Suppose that the firm is free how to allocate production between these two production functions. How does its cost function $c(y)$ look like?

Question 4 (40 points) Consider a two-good, two-consumer exchange economy. The utility functions are:

$$\begin{aligned}u_1(x_{11}, x_{21}) &= x_{11} + \sqrt{x_{21}} \\u_2(x_{12}, x_{22}) &= \sqrt{x_{12}} + x_{22}\end{aligned}$$

The initial endowments are $\omega_1 = (2, 0)$ and $\omega_2 = (0, 2)$.

(a) Characterize the set of Pareto optima and represent them in the Edgeworth box (distinguish those that are interior and those that are on the boundary).

(b) Compute the individual demand functions of the two consumers.

(c) Compute the aggregate excess demand function of this economy and show that it satisfies the gross substitute property.

(d) Show that there is a unique competitive equilibrium and find a polynomial equation that characterizes the equilibrium price of good 2 (good 1 is taken as a numeraire).

Question 5 (20 points) Consider another two-good, two-consumer exchange economy. The consumers' utility functions are:

$$\begin{aligned}u_1(x_{11}, x_{21}) &= \text{Min}\{\alpha x_{11}, x_{21}\} \\u_2(x_{12}, x_{22}) &= \text{Min}\{x_{12}, \beta x_{22}\}\end{aligned}$$

The initial endowments are $\omega_1 = (1, 0)$ and $\omega_2 = (0, 1)$.

(a) Suppose that $\alpha = 2$ and $\beta = 3$. Represent the set of Pareto optimal allocations in an Edgeworth Box. Determine the competitive price equilibrium. Is it unique?

(b) Suppose that $\alpha = \beta = 0.5$. Represent the set of Pareto optimal allocations in an Edgeworth Box. Determine all the competitive price equilibria in this Edgeworth Box.

(c) Explain the difference between the two cases.