

STUDIENZENTRUM GERZENSEE  
STIFTUNG DER SCHWEIZERISCHEN NATIONALBANK

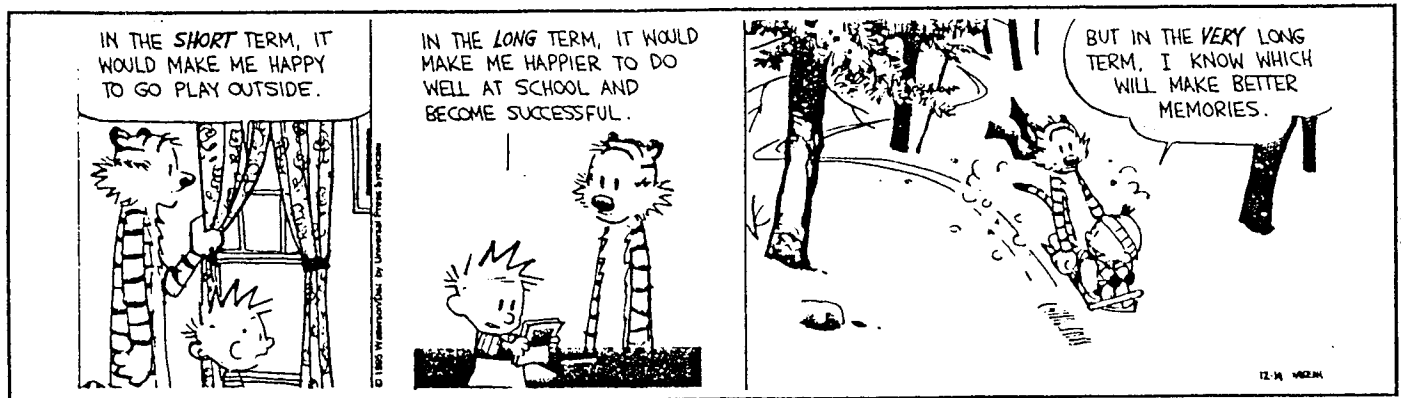
## Program for Beginning Doctoral Students in Economics 1995/96

### Exam in Macroeconomics

Tuesday, February 13, 08.30 - 10.30

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.
2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.
3. Good luck!

## CALVIN AND HOBBS



Studienzentrum Gerzensee  
Macroeconomics  
Midterm Examination

There are 120 points on this examination, which is one point per minute on a two hour exam. The 7 short answer questions are worth 7 points each, for a total of 42 points. There are three longer questions: The first longer answer question is worth 30 points; the second and third are worth 24 points each. **Please be concise and to the point in your answers; also be sure to write neatly and with large enough letters so that your answers can be read without a magnifying glass!**

**Part I: Short Answer Questions.**

1. *Forecasting with a state space model:* Suppose that aggregate output ( $y$ ) in a model economy is governed by the specification  $y_t = \pi s_t$ , where  $\pi$  is a row vector and  $s_t$  evolves according to  $s_t = M s_{t-1} + \varepsilon_t$  with  $\varepsilon_t$  an independently and identically distributed random variable that has zero mean.
  - (a) What is the forecast of  $y_{t+k}$  at date  $t$ ?
  - (b) What is the revision in the forecast of  $y_{t+k}$  that occurs as a result of  $\varepsilon_t$ ?
2. *True, False or Uncertain:* The linear stochastic difference equation,  $x_t = \rho x_{t-1} + \varepsilon_t$ , will give rise to Granger's typical spectral shape if  $\rho$  is large in absolute value.
3. *True, False or Uncertain:* In their study of U.S. business cycles, defined as deviations from a measure of trend, Kydland and Prescott found that labor and output had roughly the same volatility and that they were strongly positively correlated. Evaluate whether the following statements are true, false or uncertain (explain why).
  - (a) These facts about labor and output are largely unaffected by how one measures business cycles.
  - (b) This aspect of business cycles can be captured by RBC models, but it requires that labor respond strongly and positively to productivity shocks.
  - (c) This aspect of business cycles could easily be captured by a model without productivity shocks if there was a constant returns-to-scale production function and perfect competition.
4. *True, False or Uncertain:* If the simple model of stock prices,  $P_t = \frac{1}{1+r}(E_t P_{t+1} + E_t d_{t+1})$ , is viewed as a rational expectations model, then the condition  $r > 0$  instructs us to choose the solution that involves making the current price the present value of future dividends.
5. *True, False or Uncertain:* In the neoclassical model of investment, an unexpected increase in the date  $t$  investment tax credit will stimulate investment at date  $t$  but an expected increase in the date  $t + 1$  investment tax credit will lower investment at date  $t$ .
6. *True, False or Uncertain:* If the production function is constant returns-to-scale and takes the form,  $y_t = a_t f(k_t, n_t)$ , where  $y$  is output,  $a$  is productivity,  $n$  is labor input and  $k$  is capital, then there are the following properties of loglinear approximations near a stationary point. (A complete answer will document these properties).

- (a) Under perfect competition, the elasticity of substitution between capital and labor is irrelevant for the nature of the Solow residual.
- (b) Under perfect competition, the elasticity of substitution between capital and labor is irrelevant for the response of the real wage rate to an increase in labor or capital.

**Part II: Longer Answer Questions.**

1. *The permanent income theory of consumption:* Consider the problem of an individual who chooses consumption optimally over time. At period  $t$ , his decision problem is to maximize:

$$U_t = \sum_{j=0}^{\infty} \beta^j u(c_{t+j})$$

subject to the sequence of constraints

$$a_{t+j+1} = (1 + r_{t+j})[a_{t+j} + y_{t+j} - c_{t+j}]$$

taking as given his initial wealth  $a_t$  and the sequence of his incomes  $y_t, y_{t+1}, \dots, y_{t+j}, \dots$  as well as the paths of interest rates. We will assume that his momentary utility function takes the form  $u(c) = \frac{1}{1-\sigma}(c^{1-\sigma})$ , where  $\sigma$  is a positive parameter.

- (a) Graphically illustrate the nature of optimal choices in a two period diagram.
  - (b) What is Irving Fisher's rule for the growth rate of consumption?
  - (c) What are the first-order conditions to the dynamic optimization problem?
  - (d) Suppose that we perturb the sequence of incomes by  $\rho^j \varepsilon_t$ , which you can think of as an unexpected "shock" to the individual's income stream occurring at date  $t$ . Supposing also that the interest rate is constant through time, what is the present value of this "income shock"? How does it depend on  $\rho$ ? Draw a picture that illustrates this dependence and explain its form.
  - (e) Supposing also that  $\beta(1+r) = 1$  for all time, what will be the effect on the consumption path of the disturbance discussed in (d)? How is this outcome related to the central idea of a paper that we read during the course?
2. Consider the following version of the neoclassical model in which the savings rate is exogenous:

$$\begin{aligned} Y_t &= AK_t^{1-\alpha} \\ K_{t+1} &= I_t + (1 - \delta)K_t \\ I_t &= s(Y_t - \delta K_t) \\ Y_t &= I_t + C_t \end{aligned}$$

- (a) Characterize the steady state of this economy.
- (b) Show that the economy is globally stable.
- (c) What is the golden rule savings rate for this model?

3. Consider the following version of the neoclassical model in which the savings rate is endogenous and taxes on investment (at rate  $\tau$ ) are levied to finance government expenditures ( $G_t$ ):

$$U = \sum_{t=0}^{\infty} \beta^t \log(C_t)$$

$$Y_t = AK_t^{1-\alpha}N^\alpha$$

$$K_{t+1} = I_t + (1 - \delta)K_t$$

$$Y_t = I_t(1 + \tau) + C_t$$

The government's budget is balanced:

$$G_t = \tau I_t$$

- (a) Compute the steady state level of the capital stock for this economy.  
(b) Find the level of  $\tau$  that maximizes tax revenue in the steady state.  
(c) Suppose now that  $\delta = 1$ . Show that the optimal consumption in this economy is given by

$$C_t = \mu AK_t^{1-\alpha}N^\alpha$$

and compute the value of  $\mu$ . Is  $\mu$  a "deep structural parameter" invariant to changes in policy?