STUDIENZENTRUM GERZENSEE
STIFTUNG DER SCHWEIZERISCHEN NATIONALBANK

Program for Beginning Doctoral Students in Economics 1995/96

Exam in Microeconomics

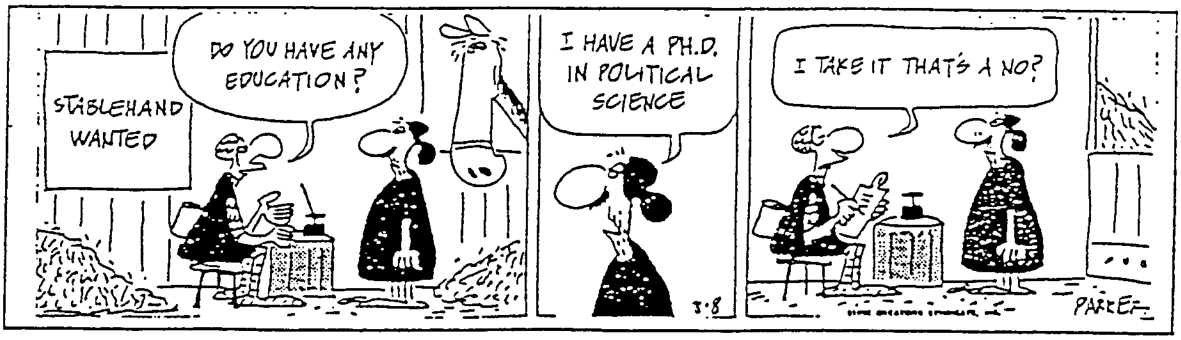
Monday, February 12, 14.00 - 16.00

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.

2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.

3. Good luck!

WIZARD of ID



Microeconomic Theory I
Studienzentrum Gerzensee
Winter 1996

Please answer all questions.

1. Use the Slutsky equation together with Cournot aggregation ($\Sigma_k p_k\, \delta g_k/\delta p_i + q_i = 0$) to show that for i=1, . . . , n

$$\Sigma_k s_{ki} p_k = 0.$$

2. Consider a linear expenditure system augmented for leisure. The utility function with one consumption good, q, and one type of leisure, *l*, is

$u = \alpha \ln(l - \underline{l}) + (1-\alpha)\ln(q - \underline{q})$,

where $\underline{l}$ denotes committed leisure and $\underline{q}$ denotes committed consumption. The expenditure function for the above LES takes the form

$C(u,p,w) = w\underline{l} + p\underline{q} + ub(p,w)$,

where $b(p,w) = w^{\alpha} p^{1-\alpha}$.

The indirect utility function takes the form

$u = \psi(w,p,X) = [X - (w\underline{l} + p\underline{q})]/b(p,w)$,

where X is full income.

a) Derive the Hicksian demand for leisure, denoted l^h.
b) Derive the Marshallian demand for leisure, denoted l^m, by substituting for u from the indirect utility function into the Hicksian demand curve.
c) Briefly explain why preferences are quasi-homothetic but not homothetic.
d) Preferences described by the above utility function are separable. Show that

$$\delta l^h/\delta p = S_{lq} = \mu(\delta l^m/\delta x)(\delta q^m/\delta x),$$

where μ is a constant which does not depend on *l* or q.
e) What does this imply about substitutability between leisure and consumption in this model? (A single sentence will suffice.)
f) How could you introduce individual heterogeneity into this model to derive an estimable labor supply equation? Assume that everyone works and that hours worked is $T - l^m$.

g) Consider a theoretical model of family utility represented by a single LES utility function, with three arguments: consumption (assumed to be a jointly consumed good), partner a's leisure and partner b's leisure. If you believed that a and b's leisure might be complements, how would you modify the basic LES utility function?

3.a) Derive the Slutsky equation for the demand for leisure and explain why the Marshallian demand for leisure does not necessarily slope downwards even when leisure is a normal good.
b) Express the Slutsky equation in elasticity form.

4. Let f(x) be the production function for a firm with a constant-returns-to-scale-technology. Suppose that each factor x_i is paid its value of marginal product, $w_i = p\delta f(x)/ \delta x_i$. Show that profits must be zero.

5. Cost functions by definition have the following properties: non-decreasing in w, homogeneous of degree 1 in w, concave in w, and continuous in w, where w is the vector of input prices. Which of the following functions satisfies these conditions? If the function is a cost function, derive the production function.
a) $c(y,w) = y^{1/2}(w_1 w_2)^{3/4}$
b) $c(y,w) = y(w_1 + (w_1 w_2)^{1/2} + w_2)$
c) $c(y,w) = y(w_1 - (w_1 w_2)^{1/2} + w_2)$

6. A farm produces corn (Y) using capital (K), labor (L) and land (T) according to the production technology described by:

$$Y = 3K^{1/3}L^{1/3}T^{1/3}$$

The firm faces prices (p,q,w,r) for (Y,K,L,T).

a) Suppose that in the short run K and T are fixed at K = T = 1. Derive the short-run supply function for the firm when w = 1.
b) Suppose that, in the long run, competitive conditions ensure that zero excess profits and further assume that w = q = 1. Furthermore, suppose labor and capital are sold on markets but land is not marketable. Derive the long-run supply function of the firm with T acres of land.
c) If p = 2, what is the long-run equilibrium output of the firm with T acres of land?
d) If there were a market for land, how much would the firm be willing to pay for more land if it is currently operating at p = w = q = 1, T = 3?

7. Consider an industry that uses two inputs, labor, L, and capital, K. Derive the own-wage elasticity of industry labor demand, η_{LL}, when production is constant returns to scale and the industry is in long run competitive equilibrium. Show that

$$\eta_{LL} = -(1-s)\sigma - s\eta$$

where $s = wL/C(w,r)y$, labor's share of total costs; $\sigma = CC_{wr}/C_wC_r$, the elasticity of substitution between labor and capital; and η is the absolute value of the own-price elasticity of demand.

Hint: The industry labor demand equation is

$$L = yC_w(w,r)$$

Market equilibrium is

$$y=D(p)$$

and

$$p = C(w,r).$$

8. What will the form of the expected utility function be if risk aversion is constant? What will be the form of the expected utility function if relative risk aversion is constant? Explain.