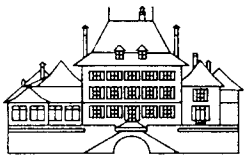

STUDIENZENTRUM GERZENSEE
STIFTUNG DER SCHWEIZERISCHEN NATIONALBANK

Program for Beginning Doctoral Students in Economics 1997

Exam in Microeconomics

Monday, August 11, 1997, 14.00 - 16.00

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.

2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.

3. Good luck!

Prof. Patricia Reagan
Ohio State University

Microeconomics Questions

1. a) Consider the case in which there are two goods, 1 and 0 . Good 0 is both an endowment and a consumption good. Good 1 is only consumed. Derive the Slutsky equation by differentiating

$$g_0(\mu + p_0T, p_0, p_1) \equiv h_0(\psi(\mu + p_0T, p_0, p_1), p_0, p_1)$$

with respect to p_0 and applying Roy's identity. The following notation applies: q_0 is the quantity consumed from the endowment of good 0. T is the total endowment of good 0. μ is exogenous income. g_0 is the Marshallian (also called Walrasian) demand function for good 0 and h_0 is the Hicksian demand function.
(Note: The uncompensated price derivative of q_0 with respect to p_0 is:
$\delta q_0/\delta p_0 \mid_\mu = \delta g_0/\delta p_0 \mid_X + \delta g_0/\delta X \, T.$)

b) Identify the substitution effect, the conventional income effect, and the revaluation-of-endowment effect.

c) Explain why the demand for consumption of the endowment, q_0 , may increase with an increase in the price, p_0 , even if good 0 is a normal good.

2. If preferences are homothetic then Marshallian (also called Walrasian) demand functions are homogeneous of degree one in wealth. Show that if preferences are homothetic and Walras' Law is satisfied, then Marshallian demand functions have unitary income elasticity?

3. In empirical work on labor supply , it is often convenient, for ease of aggregation, to assume that leisure expenditure (wage times leisure), wq_0 , is linear in w and full income, $X = wT + \mu$.

a) Show that the following expenditure function yields a leisure demand equation which, when multiplied by the wage to create the leisure expenditure function, has this property:

$$C(u,w,p) = \alpha(p) + w\beta(p) + uw^{\delta}\gamma(p),$$

where δ is a constant and p is the price of a composite consumption good.
(Hint: You must solve for u from $X = C(u,w,p)$.)

b) What homogeneity conditions must $\alpha(.)$, $\beta(.)$, and $\gamma(.)$ satisfy to insure that the above function is an expenditure function?

4. Consider a single output firm with a cost function C(y,w), where y is output and w is a vector of input prices. Show that if C(.) exhibits decreasing average cost, then C(.) is subadditive, i.e. the industry is a natural monopoly.

5. Consider a cost function

$$C(w_i, w_k, y).$$

If there are constant returns to scale firm size is indeterminant. But the industry factor demand function for input k is

$$x_k = yC_k(w_i, w_k).$$

In a competitive equilibrium

$$p = C(w_i, w_k) \text{ and}$$

$$y = D(p).$$

a) Show that the own-price elasticity of the factor demand for k is greater (in absolute value) the greater (in absolute value) is the elasticity of industry output demand.

(Hint: By successive substitutions

$$x_k = D(C(w_i, w_k))C_K(w_i, w_k).$$

Differentiate this expression with respect to w_k and substitute again to get

$$\delta x_k / \delta w_k = yC_{kk} + D'(p)C_kC_k.$$

Continue to manipulate this expression until you have an expression in terms of elasticity and factor share.)

b) Interpret each term in the final expression.

MICROECONOMICS
Andreu Mas-Colell
Gerzensee Studienzentrum
Doctoral Studies-August, 97

MIDTERM EXAM

Question 1 *(10 points).*

Consider an Edgeworth Box with strictly convex, strongly monotone and homothetic preferences. Argue that the set of Pareto optimal allocations either:

1) coincides with the diagonal, or
2) is entirely above the diagonal (except for the two end points), or
3) is entirely below the diagonal (except for the two end points)

Question 2 *(20 points)*

Consider a two-good, two-consumer exchange economy. The utility functions are:

$$u_1 (x_{11} , x_{21}) = \text{Min} \{ \alpha x_{11} , x_{21} \} \quad , \quad \alpha > 0$$
$$u_2 (x_{12} , x_{22}) = \text{Min} \{ x_{12} , \beta x_{22} \} \quad , \quad \beta > 0$$

and the endowment vectors:

$$\omega_1 = (1 , 0)$$
$$\omega_2 = (0 , 1)$$

(a) Suppose that $\alpha > 1$ and $\beta > 1$.

- Represent the set of Pareto optimal allocations in an Edgeworth Box.
- Determine the Walrasian price equilibrium. Is it unique?

(b) Suppose that $\alpha < 1$ and $\beta < 1$

- Represent the set of Pareto optimal allocations in an Edgeworth Box.
- Determine all the Walrasian price equilibria in this Edgeworth Box.

Question 3 *(20 points)*

We have an economy with two goods: a numeraire and a consumption good.

There are I consumers with identical utility functions:

$$u_i(x_i, m_i) = \text{Min}\ \{ x_i, (1/2)\, x_i + (1/2)\ \} + m_i$$

Denote by p the price of the consumption good.

(a) Determine precisely and represent graphically the demand function (or correspondence) of this economy.

Suppose now that there is a number J of identical firms that may produce consumption good out of numeraire. The cost function is

$$c(q_j) = (\alpha/2)\, q_j^2$$

We assume that there is the same number of consumers as of firms, that is, I = J.

(b) For every value of the cost parameter $\alpha = 0$ determine the Walrasian equilibrium price and productions of this economy (In particular, note that profit shares are immaterial and that equilibrium is unique).

Question 4 *(10 points)*

There are two types of consumers. We know from an exercise that a core allocation of an exchange economy with two consumers of one type and one of the other need not enjoy the equal treatment property. Show, however, that if the economy has four consumers of one type and two of the other then every core allocation has the equal treatment property.