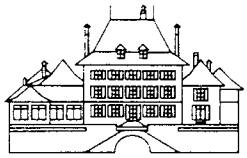

STUDIENZENTRUM GERZENSEE
STIFTUNG DER SCHWEIZERISCHEN NATIONALBANK

Swiss Program for Beginning Doctoral Students in Economics 1998

Exam in Macroeconomics

Tuesday, July 28, 1998, 09.00h - 11.00h

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.

2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously. Please use **a pen** rather than a pencil so that your answers can be read without problems.

3. Good luck!

ID-Number:_____

ID Number:

Studienzentrum Gerzensee
Macroeconomics
Midterm Examination

There are 120 points on this examination. The 5 short answer questions are worth 8 points each, for a total of 40 points. There are four longer questions: each is worth 20 points, for a total of 80 points. For multiple part questions, the individual components (a),(b) and so forth are weighted equally within questions.

Please be concise and to the point in your answers; also be sure to write neatly using a dark pen and with large enough letters so that your answers can be read without a magnifying glass!

Part I: Short Answer Questions.

1. *Properties of the univariate state space model:* Suppose that aggregate output (y) in a model economy is governed by the specification $y_t = \pi s_t$, where π is a row vector and s_t evolves according to $s_t = M s_{t-1} + \varepsilon_t$ with ε_t an independently and identically distributed random variable that has zero mean.

   (a) What is the forecast of y_{t+k} at date t?

(b) What is the revision in the forecast of y_{t+k} that occurs as a result of ε_t?

(c) If ρ is close to one, what will the spectrum of y look like? How will it depend on π?

2. *The cyclical relationship of labor and output:* If one looks at U.S. labor and output as deviations from a linear trend, then output and labor are not too highly correlated. But, if one looks at these same series after they have been filtered using the Hodrick-Prescott filter, then these series are very highly correlated. Use your knowledge of frequency domain material to answer the following questions about this fact.

   (a) Suppose that we divide output and labor into components, $y_t = y_t^c + y_t^o$ and $n_t = n_t^c + n_t^o$ where a superscript 'c' denotes the HP cyclical component and the superscript 'o' denotes the remainder. Why does the frequency domain analysis and the properties of the HP filter suggest that the cyclical and other components of output will be approximately uncorrelated?

(b) If labor and output are only weakly correlated as deviations from trend, what does this suggest about the statistical properties of y_t^o and n_t^o?

3. *True, False or Uncertain:* If the simple model of stock prices, $P_t = \frac{1}{1+r}(E_t P_{t+1} + E_t d_{t+1})$, is viewed as a rational expectations model, then the condition $r > 0$ instructs us to choose the solution that involves making the current price the present value of future dividends.

4. *True, False or Uncertain:* In the basic indivisible labor model, there is an infinitely elastic labor supply if the marginal utility of consumption is held fixed.

5. From the perspective of the neoclassical model of investment, the implicit rental price of a new car for a single year involves the cost of purchasing the car, the market value of the used car at year's end and the relevant interest rate. Briefly indicate how each of these components affects the implicit rental price.

Part II: Longer Answer Questions.

1. *The permanent income theory of consumption:* Consider the problem of an individual who chooses consumption optimally over time. At period t, his decision problem is to maximize:

$$U_t = \sum_{j=0}^{\infty} \beta^j u(c_{t+j})$$

subject to the sequence of constraints

$$a_{t+j+1} = (1 + r_{t+j})[a_{t+j} + y_{t+j} - c_{t+j}]$$

taking as given his initial wealth a_t and the sequence of his incomes $y_t, y_{t+1}, ...y_{t+j}, ...$ as well as the paths of interest rates. We will assume that his momentary utility function takes the form $u(c) = \frac{1}{1-\sigma}(c^{1-\sigma})$, where σ is a positive parameter.

(a) Graphically illustrate the nature of optimal choices in a two period diagram.

(b) What is Irving Fisher's rule for the growth rate of consumption?

(c) What are the first-order conditions to the dynamic optimization problem?

(d) Suppose that we perturb the sequence of incomes by $\rho^j \varepsilon_t$, which you can think of as an unexpected 'shock" to the individual's income stream occurring at date t. Supposing also that the interest rate is constant through time, what is the present value of this 'income shock"? How does it depend on ρ? Draw a picture that illustrates this dependence and explain its form.

(e) Supposing also that $\beta(1+r) = 1$ for all time, what will be the effect on the consumption path of the disturbance discussed in (d)?

2. *The Solow Model with Exogenous Savings.* Consider the following version of the neoclassical growth model with exogenous savings:

$$Y_t = F(K_t, N_t)$$

$$I_t = sY_t$$

$$K_{t+1} = (1 - \delta) K_t + I_t$$

(a) Characterize the steady-state level of per capita capital, k. What property, or properties, of $F(.)$ guarantee that the steady state is unique?

(b) How does the steady state level of capital stock change in response to a decrease in the rate of savings, s?

(c) Characterize the golden-rule level of capital. What is the real interest rate associated with the golden-rule?

(d) Characterize graphically the global stability of the system. What property or properties of $F(.)$ guarantee global stability?

3. *A Stochastic Version of the Solow Model.* Consider the following stochastic version of the Solow model:

$$K_{t+1} = sAK_t^{1-\alpha} + (1 - \delta_t) K_t$$

where δ_t is an i.i.d. random variable that can take on two values, δ_L and δ_H ($\delta_L < \delta_H$) with equal probability and that has mean $\underline{\delta}$.

(a) Linearize this model around the steady state.

(b) Use the linearized solution to characterize the stationary (limiting) distribution of the capital stock.

(c) What is the golden rule savings rate for this model?

4. *Singular linear systems analysis of a perfect foresight model.* Consider the following equations that describe the approximate dynamics of the fixed labor neoclassical model near its steady state:

$$\begin{aligned} c_t &= -\sigma\lambda_t \\ k_{t+1} &= mk_t - \phi c_t \\ \lambda_{t+1} - \eta k_{t+1} &= \lambda_t \end{aligned}$$

where the first of these equations describes how consumption is related to its shadow price, the second how capital accumulates, and the third indicates how the evolution of the shadow price is related to the future stock of capital. All parameters are assumed positive, although the answers to the questions below do not use this fact.

(a) Defining the column vector $y_t = [c_t \ \lambda_t \ k_t]'$, write this model in the form $Ay_{t+1} = By_t$, where A and B are 3x3 matrices of coefficients. Show that the matrix A is singular.

(b) Which variables of the model are predetermined? Which are not?

(c) Use the first equation to substitute out for consumption, leading to a two variable system in the column vector $d_t = [\lambda_t \; k_t]'$. This system will take the form $\alpha y_{t+1} = \beta y_t$, where α and β are 2x2 matrices of coefficients. Show that the matrix α is not singular.

(d) There are two eigenvalues of the matrix $w = \alpha^{-1}\beta$, which you need not derive. Based on your knowledge of real business cycle models and rational expectations models, how many stable and unstable eigenvalues of w should there be?

(e) (experts only, not for credit). Show that the finite solutions to $|Az - B| = 0$ and the solutions to $|Iz - w| = 0$ are identical.|