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Midterm Examination 1999
Macroeconomics

There are 120 points on this examination, which is roughly one point per minute over two hours. Part A of the exam contains short questions for a total of 30 point: there are 5 short questions worth 6 points each. These are all of the "true, false or uncertain form" or TFU for short: your job is to explain why the assertion is best associated with one of these three descriptions, with all credit based on this explanation. Part B of the exam contains three longer questions, which are each worth 30 points. When there are multiple parts to these longer questions, the parts are equally weighted within each question.

A. Short Answer Questions (30 points total)

(1) TFU According to neoclassical investment theory, an increase in the interest rate or the rate of depreciation will lower the optimal level of the capital stock.

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(2) TFU An overlapping generations model is always dynamically efficient, given that each generation maximizes its lifetime utility.

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(3) TFU Global stability always implies local stability

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(4) TFU In the Hansen-Rogerson indivisible labor model, there is an infinite "Frisch" labor supply elasticity for the stand-in representative agent. However, when this specification is embedded into an RBC model, there will be no long-run effect of an increase in labor productivity on the quantity of employment.

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(5) TFU Suppose that stock prices and dividends are described by the following system

$$\begin{bmatrix} 0 & 0 \\ 1+r & 1+r \end{bmatrix} E_t \begin{bmatrix} d_{t+1} \\ p_{t+1} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d_t \\ p_t \end{bmatrix} + \begin{bmatrix} c_{d1} & c_{d2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix}$$

This system has a single finite eigenvalue, which is equal to $(1+r)$. Its solution can be written as

$$\begin{aligned} d_t &= c_{d1}x_{1t} + c_{d2}x_{2t} \\ p_t &= \sum_{j=1}^{\infty}(\frac{1}{1+r})^j E_t[c_{d1}x_{1,t+j} + c_{d2}x_{2,t+j}] \end{aligned}$$

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B. Longer Questions (90 points total, 30 points per question)

1. *Optimal consumption over time*

Consider a household which has an exogenous income stream, which takes the form

$$\begin{aligned} y_t &= \overline{y} \text{ for } t = 0, 2, 4, ... \\ y_t &= \underline{y} \text{ for } t = 1, 3, 5, ... \end{aligned}$$

Assume that it can borrow or lend at interest rate r_t, so that its wealth accumulates according to

$$a_{t+1} = (1 + r)[a_t + y_t - c_t]$$

but that it starts at date 0 with an asset balance of zero.

The household is concerned with its life-time utility,

$$\begin{aligned} U &= \sum_{t=0}^{T} \beta^t u(c_t) \\ u(c) &= \frac{1}{1-\sigma}[c^{1-\sigma} - 1] \end{aligned}$$

It must end life with a nonnegative asset balance, i.e., $a_{T+1} \geq 0$.

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(a) Write down a dynamic optimization problem for the household and display the first-order conditions. Why will these conditions be sufficient for an optimum, as well as necessary?

(b) Suppose that the interest rate is constant through time and that $\beta(1+r) = 1$. Derive the optimal consumption plan. How does it depend on σ? Why? How will even periods be different from odd periods for this household?

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(c) Suppose that the interest rate is not constant through time. Derive the optimal consumption plan. How does it depend on σ? Why?

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(d) There is some sequence of interest rates which will make the household not want to borrow or lend in any period. What is it?

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2. *The Neoclassical Growth Model*

Consider the following planner's problem:

$$\max U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \log(c_t - \psi n_t^{\theta}) \right\}$$

s.t.

$$k_{t+1} = A_t k_t^{\alpha} n_t^{1-\alpha} - c_t$$

where $\log(A_t)$ follows a stationary AR(1) process.

(a) Derive the first order conditions for this problem.

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(b) Assume, for the moment that A_t is constant and characterize the steady state of this economy

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(c) Show that a solution to the stochastic problem has the form $c_t = \mu A_t k_t^{\alpha} n_t^{1-\alpha}$ and

compute μ.

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(d) Discuss whether this model can replicate the main stylized facts of business cycles.

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3. *Overlapping Generations*

Consider the following simple overlapping generations model in which each individual lives for two periods. Total population is constant and normalized to 1. Every individual supplies one unit of labor inelastically when young earning the wage w_t, and saves $s_t \geq 0$ in the first period to obtain $(1 + r_{t+1}) s_t$ the following period. There are no other endowments. Therefore, an individual of generation $t \geq 1$ solves:

$$\max \log(c_{1t}) + \beta \log(c_{2t+1})$$

s.t.

$$c_{1t} + s_t \leqq w_t$$
$$c_{2t+1} \leqq (1 + r_{t+1}) s_t$$

Assume that the first generation is old, owns $s_0 > 0$ of savings and cares only about its consumption.

Firms in this economy solve

$$\max \; AK_t^{1-\alpha} N_t^{\alpha} - w_t N_t - (1 + r_t) K_t$$

(a) Derive the optimal savings rule for the representative consumer of generation t.

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(b) Derive the firm's input demand functions.

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(c) Derive the equilibrium law of motion for capital for this economy.

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(d) Characterize the steady state of this economy.