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Midterm Examination 1999
Microeconomics

Answer all questions. There are 100 points.

1. (20 points) Consider a two product firm with demand functions

$$Q_1 = 40 - 2P_1 - P_2$$

$$Q_2 = 35 - P_1 - P_2$$

and a cost function

$$C = Q_1^2 + 2 Q_2^2 + 10.$$

a) Find the output levels that satisfy the necessary conditions for maximum profit.
b) Check the satisfaction of the second order conditions.
c) What is the maximal profit?

2. (20 points) Consider a utility function

$$x^{\varepsilon}y^{1-\varepsilon},$$

with $0<\varepsilon<1$ and a linear budget constraint. Show that the Marshallian demand functions violate homogeneity if $\varepsilon=\varepsilon(p_x)$, but do not violate homogeneity if $\varepsilon=\varepsilon(p_x/ p_y)$.

3. (30 points) Consider an indirect utility function of the form

$$u(w,p) = a(p) + b(p)w,$$

where w is wealth and p is a price vector.

a) What characteristics must the functions a(p) and b(p) display?
b) Show that the wealth elasticity of demand for the kth good is constant.
c) Graph the income expansion paths, i.e., Engel curves.
d) If preferences are homothetic, what additional restriction would be placed on the indirect utility function?

4. (30 points)
a) Consider the case in which there are two goods, 1 and 0 . Good 0 is both an endowment and a consumption good. Good 1 is only consumed. Derive the Slutsky equation by differentiating

$$g_0 (\mu + p_0 T, p_0 , p_1) \equiv h_0 (\psi (\mu + p_0 T, p_0 , p_1), p_0 , p_1)$$

with respect to p_0 and applying Roy's identity. The following notation applies: q_0 is the quantity consumed from the endowment of good 0. T is the total endowment of good 0. μ is exogenous income. g_0 is the Marshallian (also called Walrasian) demand function for good 0 and h_0 is the Hicksian demand function.
(Note: The uncompensated price derivative of q_0 with respect to p_0 is:
$\delta q_0/\delta p_0 |_{\mu} = \delta g_0/\delta p_0 |_X + \delta g_0/\delta X$ T.)

b) Identify the substitution effect, the conventional income effect, and the revaluation-of-endowment effect.

c) Explain why the demand for consumption of the endowment, q_0, may increase with an increase in the price, p_0, even if good 0 is a normal good.

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Question 1 (30 points)

Consider an economy with two consumers, one firm, and two goods: leisure l and consumption C. Consumers have identical utility functions: $u(C, l) = C^a l^{(1-a)}$, and consumer 2 possesses the firm. They also have an endowment of T units of time, which they can allocate between leisure l and work L. They are no initial endowments in the consumption good.

1. Compute the labor supply of agent 1 as a function of the wage rate w (the consumption good is taken as a numeraire).

2. Compute the labor supply of agent 2 as a function of the wage rate w and of the profit π of the firm.

3. The production function of the firm is $q = Log(L)$. Compute its labor demand and profit functions.

4. Determine the equilibrium wage rate w as a function of the parameters a and T .

5. The government imposes to the firm a tax on labor at rate t, and redistributes the receipts to consumer 2. Study the impact on the equilibrium wage rate and allocations. What would be the impact of a tax on profits ?

Question 2 (20 points)

Consider an Arrow Debreu economy with two dates ($t = 0, 1$) and three states ($s = 1, 2, 3$) at date 1. There are four securities:

- A stock, with a price S at date 0 and a liquidation value R_s in each state s at date 1.
- A bond, with a price B at date 0 and a liquidation value 1 in each state s at date 1.
- A call option on the stock, with a price C at date 0 and a liquidation value $Max(0,\ R_s - K)$ in each state s at date 1.
- A put option on the stock, with a price P at date 0 and a liquidation value $Max(0, K - R_s)$ in each state s at date 1.

1. Recall under what assumptions it is possible to find three numbers $\mu_s \geq 0$ such that the price of each security is equal to the sum of its liquidation values in all future states multiplied by the μ_s.

2. Show that this implies the following relation $C - P = S - KB$

3. Explain why C cannot be deduced exactly from S, B and R_s.

4. Show however that C has to satisfy certain inequalities. What is the maximum value of C that satisfies these inequalities?