The Option Value of Money*

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Abstract

An intrinsically useless asset can have value not because it serves as a medium of exchange today, but because it could in the future— this is the option value of money. We characterize the private and social option values of cryptocurrency in a model with a possibly self-interested government controlling the cash supply. The social value is ambiguous: negative because cryptocurrency raises the cost of holding cash, yet positive when it disciplines the government. Calibrating the model, we find households would forgo 0.10%-0.81% of consumption to live in an economy where Bitcoin carries option value.

Keywords: cryptocurrency, money demand, asset price, inflation

JEL Codes: E31, E41, E42, E44

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1 Introduction

Technological advances—notably in cryptography and computing—have enabled the creation of new forms of money, such as Bitcoin and Ether. Yet many policymakers and lawmakers, as well as some academics, seriously question whether these technologies generate any social value. The argument typically runs as follows: cryptocurrencies neither pay dividends nor are widely used as a medium of exchange—except in illegal transactions—and are therefore purely speculative assets that serve no societal function. We challenge this view by arguing that seemingly speculative assets—even if intrinsically useless and devoid of non-pecuniary functions such as liquidity provision—may nevertheless serve an important social role.

To make this point, we first develop a theory of why intrinsically useless money may be privately valued. The core idea is simple and intuitive, yet has not been formally modelled: money can have value not because it serves as a medium of exchange *right now*, but because it *might* do so in the *future*. We call this the *private option value* of money.[1] The existence of an equilibrium with a positive private option value is not obvious: if agents are willing to accept an asset as a means of payment tomorrow, should they not accept it today as well? Second, we examine the *social option value* of money, that is, the social value of a useless object that has no role today, but is a possible payment device in the future. We show the social option value may be positive, by disciplining the government, or negative, by raising the opportunity cost of holding current media of exchange. Third, we apply this theory to cryptocurrency, with a focus on Bitcoin, and provide an empirical estimate of the magnitude of the social option value in practice.

In order to address these questions, we build on the model by Lagos and Wright (2005) and more specifically Lester et al. (2012), because we need a tractable model in which money is both essential and its use arises endogenously.[2] There are buyers and sellers who sometimes trade in

[1] The terminology is motivated by analogy with financial options: just as a call option derives value from the possibility of being exercised in the future, crypto derives value today from the possibility—but not the certainty—of being used as a medium of exchange in the future.

[2] This rules out most New Keynesian models which either abstract from money altogether or introduce it in a reduced-form manner (Lagos and Zhang (2022)); it also rules out cash-in-advance models (Clower (1967)) and money-in-the-utility-function models (Sidrauski (1967)), in which the use of money is imposed by assumption rather than derived endogenously. Furthermore, while sticky prices could in principle be accommodated, fully flexible prices are a natural benchmark: they simplify the analysis and ensure that the option value mechanism

anonymous bilateral matches—generating a need for a medium of exchange—and sometimes in non-anonymous Walrasian markets—allowing agents to rebalance their asset portfolios.[3] There are two intrinsically useless assets that may endogenously serve as media of exchange: government money, henceforth "cash", and a second asset, henceforth "crypto".In the simplest version of the model there is only one difference between cash and crypto: while sellers can always accept cash as payment, they can only accept crypto if they have paid the necessary adoption cost (e.g., the cost of acquiring a payment device or the effort to learn how to handle cryptocurrency).

In this context, we introduce a random sunspot that occurs at most once. We then construct an equilibrium in which crypto has option value. Specifically, before the sunspot occurs, buyers and sellers trade exclusively with cash, sellers do not adopt the crypto technology, yet crypto has positive value and buyers hold it. Sellers adopt the crypto technology when the sunspot occurs, and it is henceforth used as a medium of exchange.

We characterize the conditions under which this equilibrium exists and we establish the following properties. First, such an equilibrium does not generically exist. Specifically, the adoption cost of the crypto technology must be neither too high—in which case sellers would never adopt it—nor too low—in which case sellers would adopt it immediately. Second, the pre-sunspot value of crypto adjusts so as to generate a sufficiently high expected return to induce agents to hold it purely as a speculative asset. This occurs precisely because crypto is expected to be adopted as a medium of exchange with positive probability and is therefore expected to appreciate.Moreover, we derive a closed-form pricing formula for crypto that depends positively on its future liquidity value and the adoption probability. Third, the presence of an asset with option value worsens the allocation, conditional on both currencies being otherwise identical (i.e., growing at the same rate). The reason is that the value of cash falls when agents replace it with crypto, hence making it more costly to hold before the adoption of crypto takes place and thereby hampering its current function as the sole medium of exchange. We refer to this as "the replacement effect".

Although informative for exposing the option value mechanism, the simple model is too stylized to bring to the data. Moreover, it abstracts from the fact that the adoption of a

is not obscured by independent sources of monetary non-neutrality.

[3]This two-market structure offers multiple advantages, as discussed in Lagos and Wright (2005). For our purposes, the most notable is that bilateral meetings with explicit bargaining allow for a more natural treatment of off-equilibrium deviations—something that will be crucial in our analysis.

competing currency rarely occurs without a fundamental reason, but is instead typically driven by a deterioration of the value of the incumbent currency. This observation also motivates a key distinction between crypto and cash so far absent from the model: whereas the supply of the latter is determined by the government and is therefore subject to abuse, the former is algorithmically controlled and immune to such manipulation.

In an extended model, we therefore replace the sunspot with the event that the government turns "rogue": rather than setting the growth rate of cash so as to maximize welfare, a rogue government maximizes its own consumption by extracting seigniorage revenue through the inflation tax. The growth rate of cash is thus endogenously determined by the government's program, while the growth rate of crypto is, by contrast, fixed. We further enrich the model with additional features—such as a convex transaction cost for crypto—that allow us to match key moments in the data. We again focus on equilibria in which crypto acquires option value, but also characterize equilibria in which crypto is only partially adopted—that is, adopted by a fraction of sellers.

Relative to the simple model, three new conceptual insights emerge. First, the presence of crypto with option value disciplines the government: if the government inflates the money supply too aggressively, buyers will switch exclusively to crypto, eliminating the seigniorage base entirely. We refer to this as the *disciplining effect*. Second, this disciplining effect may be sufficiently strong to outweigh the replacement effect—in which case the cost of holding cash may actually *fall*, raising consumption even before crypto is adopted.[4]

In order to quantify the social option value of crypto—the welfare gain attributable to the presence of a crypto option value—we calibrate the model to the US economy, taking Bitcoin as the relevant crypto currency. Two findings stand out. First, the annual adoption probability implied by the Bitcoin price lies between 0.20% and 2.96%. Second, transaction costs induced by Bitcoin payments are small relative to aggregate economic activity—below 0.35% of GDP—even in a scenario where most transactions are conducted with Bitcoin.

All calibrations suggest that the social option value of Bitcoin is positive, ranging between

[4]These insights hold unconditionally only when *all* sellers adopt crypto—so-called full-adoption equilibria. Under some parameters, however, partial-adoption equilibria also exist, where only a fraction of sellers adopt crypto because sellers are indifferent between adopting and not. In such equilibria, a "reverse disciplining effect" may arise. We argue that this case is empirically less relevant but study it nevertheless in Appendices B and C.

0.10% and 0.81% in consumption equivalents. Although the probability that the "option is exercised" is low, the model implies that when it is, the disciplining effect is substantial: consumption under crypto adoption is approximately seven times larger than in an equilibrium without crypto. This follows from two features of the calibration: (a) optimal inflation is high in the absence of crypto, ranging between 46% and 76% annually; and (b) the disciplining effect is strong, given the low transaction costs of Bitcoin and the essentially fixed long-run supply of Bitcoin. The effect is sufficiently strong that consumption rises even before the government turns rogue—albeit only marginally. At an average annual household consumption of $75,000, agents would be willing to pay between $75 and $600 per year to live in a world where Bitcoin has option value.

Literature review. Our basic framework builds on the New Monetarist tradition—most notably Kiyotaki and Wright (1989), Trejos and Wright (1995), and Lagos and Wright (2005)—that microfounds the demand for money. See Lagos et al. (2017) for an overview of the literature. More specifically, we build on Lester et al. (2012), who study an economy in which buyers may use currencies that sellers do not necessarily recognize.

Cash and crypto. Many papers look at the asset pricing implications of crypto monies. For example, Schilling and Uhlig (2019), Benigno et al. (2022) and Uhlig (2024) study the equilibrium properties of an economy where cash and crypto money are both valued. Recent dynamic models of crypto adoption and pricing include Cong et al. (2021) and Sockin and Xiong (2023). But the asset pricing approach takes the existence of crypto as given and studies the pricing implications on other assets.

In a similar vein, the search-theoretic approach of Choi and Rocheteau (2021, 2022) embed Bitcoin in a New Monetarist environment with endogenous mining and characterize price dynamics under the assumption that the cryptocurrency is *actively* used as a medium of exchange in equilibrium. Somewhat differently, Altermatt et al. (2026) point out that crypto can be valued when it is used as collateral rather than as a direct means of payment. We depart from these papers by showing that crypto does not have to currently serve as a means of payment (or as collateral) to be valued—it suffices that it *may* do so in the future. This yields two distinct results. First, on the positive side, our framework can rationalize the well-documented disconnect between Bitcoin's market capitalization and its transactional throughput—highlighted by

Yermack (2015), Baur et al. (2018), and Lammer et al. (2020). Second, on the normative side, the welfare effects of crypto in our model operate not through realized transactional efficiency, as in Choi and Rocheteau (2021, 2022), but through the equilibrium constraint that the *mere existence* of the option imposes on the incumbent monetary authority—a disciplining channel absent in the equilibrium pricing model of Biais et al. (2023).

The two papers closest to ours are probably Choi and Liang (2023), and Biais et al. (2025). Both papers study an economy with crypto that is valued in equilibrium because it is rendering immediate transactional or saving services to some set of agents. Choi and Liang (2023) considers an economy where agents gradually learn about the holding cost of a new asset which can be adopted or not as a means to pay. In their set up, agents learn about the properties of a partially liquid currency, while in our model all properties are known and crypto is valued although it can be illiquid. In contrast, Biais et al. (2025) has no learning, but study the disciplining role of crypto on a non-benevolent government, very much like our equilibrium once the government turns rogue. So their analysis differ from ours in two respects: they focus on money as a safe store of value rather than as a medium of exchange, and they consider the actions of a rogue government only. In that sense, our papers are complementary.

In a different context, Lagos and Zhang (2022) show that money can affect the equilibrium allocation as a latent, never-exercised, option as a medium of exchange, when transactions are otherwise settled through credit. Our mechanism shares the same logical structure—value derived from a contingent monetary role—but applies it to the disciplining of an incumbent sovereign currency by an entrant cryptocurrency

Currency competition and coexistence. We also share intellectual ground with the currency-competition tradition, going back to at least Von Hayek (1976). Fernández-Villaverde and Sanches (2019), Schilling and Uhlig (2019), Benigno et al. (2022), and Guennewig (2026) are more recent contributions that focus on competition between currencies that are *simultaneously* circulating, whereas our mechanism operates precisely when they are not.

A long tradition studies when two distinct objects can simultaneously serve monetary functions. Wallace (1981) established the overlapping generations model as a model of fiat money (also see Wallace (2001)), with Kareken and Wallace (1981) establishing the canonical exchange-rate-indeterminacy result for two intrinsically useless fiat monies. In search-theoretic environments, Zhou (1997), Curtis and Waller (2000), Head and Shi (2003), and Craig and Waller (2000) characterize when two currencies coexist, how their use specializes across transactions, and how their exchange rate is determined. Closest in spirit to our setup, Lotz and Rocheteau (2002) and Lotz (2004) analyze the launch of a new currency alongside an incumbent—an exercise we extend by allowing for the possibility that adoption materializes only *stochastically ex post*, yet the new asset commands a positive equilibrium price *ex ante* through its option value. Extending the indeterminacy logic to privately issued outside monies, Garratt and Wallace (2018) provide an antecedent for our cryptocurrency analysis. Finally, the literature on central bank digital currency—notably Brunnermeier and Niepelt (2019) and Keister and Sanches (2023)—studies coexistence between public and private monies and, in line with our disciplining-government channel, considers how the availability of alternative payment instruments constrains the issuers of inside money.

Outline. The remainder of this paper is structured as follows. Section 2 introduces the basic model and the notion of private option value. Section 3 presents a richer model that incorporates crypto-specific features and an endogenous government determining monetary policy. Section 4 calibrates the model using US and Bitcoin data and estimates the social option value of crypto. Section 5 concludes.

2 A Simple Model

Time $t = 0, 1, \dots \infty$ is discrete and goes on forever. A continuum of *buyers* and *sellers*, with measure one each, discount the future at rate $\beta \in (0, 1)$. Each period consists of two sub-periods. In the first sub-period, a *decentralized market* (DM) opens where agents trade the DM-good. In the second sub-period, agents can trade the CM-good in a *centralized market* (CM). Goods are not storable across markets or across periods.

In the DM, buyers derive utility $u(q) : \mathbb{R}_0^+ \to \mathbb{R}$ from consuming $q \geq 0$ units of the DM-good, where $u(q)$ is increasing and concave with $u(0) = 0$, $\lim_{q \to +\infty} u'(q) = 0$ and $\lim_{q \to 0} u'(q) = +\infty$. Sellers suffer disutility $c(q) : \mathbb{R}_0^+ \to \mathbb{R}$ from producing DM-goods, where $c(q)$ is increasing and weakly convex with $c(0) = 0$. There is a unique $q^* > 0$ that maximizes the trade surplus, defined as $\mathcal{S}(q) \equiv u(q) - c(q)$, and satisfies $u'(q^*) = c'(q^*)$.

In the CM, buyers and sellers derive linear utility x from consuming $x \geq 0$ units of the CM-good. All agents suffer disutility y from producing CM-goods.

Between the DM and the CM, a public sunspot—possibly serving as a coordination device—occurs with probability $\rho \in [0, 1)$ and at most once. Let $s_t \in \{0, 1\}$ denote the aggregate state at time t, where $s_t = 1$ if the sunspot has occurred before period t and $s_t = 0$ otherwise. The state follows a Markov chain: $s_{t-1} = 0$ transitions to $s_t = 1$ with probability ρ, state $s_t = 1$ is absorbing, and the initial condition is $s_0 = 0$. For any function $x(s_t)$, the one-step-ahead expectation $\mathbb{E}_{s_t|s_{t-1}}[x(s_t)]$ equals $\rho\, x(1) + (1 - \rho)\, x(0)$ if $s_{t-1} = 0$ and $x(1)$ if $s_{t-1} = 1$. The expected lifetime utility of buyers and sellers is, respectively,

$$\mathcal{U}^B = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_{s_t|s_{t-1}} \Big[(u(q_t(s_{t-1})) + x_t(s_t) - y_t(s_t)) \Big],$$

$$\mathcal{U}^S = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_{s_t|s_{t-1}} \Big[(-(q_t(s_{t-1})) + x_t(s_t) - y_t(s_t)) \Big].$$ [5]

In the DM, buyers and sellers are randomly matched with probability $\sigma \in (0, 1]$. We set $\sigma = 1$ for the remainder of this section but allow for $\sigma \leq 1$ in Section 3, where it helps match the data.Because buyers are anonymous and cannot commit to repay sellers in the subsequent CM, trade in the DM requires a medium of exchange.[6] Two assets can serve that role: *crypto currency* (C), and *government currency / cash* (G). The supply of currency $c \in \{G, C\}$ evolves according to $M_{t+1}^c = \gamma^c M_t^c$, where $\gamma^c \in [1, \infty)$ denotes the gross growth rate with $M_0^c > 0$.[7] Its value in terms of the CM-good is $\phi_t^c(s) \in \mathbb{R}_0^+$, in sunspot state s. The stock of currency c evolves through lump-sum transfers to all buyers in the CM denoted by $T_t^c = M_{t+1}^c - M_t^c$.[8]

We assume that while sellers can always accept cash in DM transactions, they can only accept crypto if they adopt the "crypto technology."[9] Adopting the technology for the DM in period t

[5]Because the sunspot occurs after the DM but before the CM, DM consumption q_t is a function of s_{t-1} while CM consumption x_t and CM production y_t are a function of s_t.

[6]See Isch-Taudien and Schneider (2026) for a detailed analysis of anonymity, credit, and money in such environments.

[7]Implicitly, we assume that money supplies can not fall, reflecting that neither crypto issuers nor central banks can levy taxes in our economy. This is not crucial for the argument so long as currency is costly to hold, i.e. we are away from the Friedman Rule.

[8]The distribution of transfers between sellers and buyers is unimportant for our analysis.

[9]Explicit microfoundations for this would be straightforward to add. For example, following Lester et al. (2012), suppose buyers could costlessly counterfeit cryptocurrency in the DM; the crypto technology can then be interpreted as enabling sellers to verify genuine crypto, without which they would not accept it.

incurs a cost $\chi \in \mathbb{R}^+$ to be paid in the CM in $t-1$. We assume that adoption is not permanent.[10] Let $\delta_t \in [0,1]$ denote the share of sellers who adopted the technology in the CM of period $t-1$.

Recursive Equilibrium

Next, we characterize the equilibrium in recursive form. Whenever no confusion arises, we suppress the explicit dependence of variables on t and s to simplify notation.

Centralized Market

A buyer entering the CM in state s with currency holdings (m^G, m^C) has value function:

$$W_s^B(m^G, m^C) = \max_{x,y,m_{+1}^G,m_{+1}^C} x - y + \beta V_s^B(m_{+1}^G, m_{+1}^C)$$
$$\text{s.t. } x + \phi_s^G m_{+1}^G + \phi_s^C m_{+1}^C = y + \phi_s^G m^G + \phi_s^C m^C + \tau_s,$$

where $\tau_s = \phi_s^G T_s^G + \phi_s^C T_s^C$ is the total real value of transfers, and $V_s^B(m_{+1}^G, m_{+1}^C)$ is the next period's DM value function in state s for a buyer leaving the CM with currency holdings (m_{+1}^G, m_{+1}^C). Similarly, for sellers we can write:

$$W_s^S(m^G, m^C) = \phi_s^G m^G + \phi_s^C m^C + \max_{m_{+1}^G, m_{+1}^C, e} -\left(\phi_s^G m_{+1}^G + \phi_s^C m_{+1}^C\right) - e_{+1}\chi + \beta V_s^S(m_{+1}^G, m_{+1}^C, e_{+1}), \tag{1}$$

where we have used the seller's budget constraint to eliminate x_t and y_t. Furthermore, $e_{t+1} \in \{0,1\}$ is the *adoption choice* of sellers allowing them to accept crypto in the next period's DM. Sellers adopt if the benefit of accepting crypto in the next DM exceeds the adoption cost χ:

$$\beta\left[V_s^S(m_{+1}^G(1), m_{+1}^C(1), 1) - V_s^S(m_{+1}^G(0), m_{+1}^C(0), 0)\right] \geq \chi,$$

where $m_{+1}^c(e_{+1})$ denotes optimal holdings of currency c given adoption choice e_{+1}. Sellers do not adopt if the inequality is reversed, and they are indifferent otherwise.

Going forward, let us denote the sellers' holdings of currency c by $\tilde{m}_t^c$. Following Lagos and Wright (2005), by inserting the budget constraints of both programs into their respective objective functions, we can immediately see that money holdings enter linearly into the CM value

[10] Alternatively, we could assume permanent adoption; given our focus on stationary equilibria, this distinction is irrelevant and the assumption merely simplifies some expressions.

function. The first-order conditions (FOCs) with respect to currency $c \in \{G, C\}$ for buyers and sellers are, respectively,

$$-\phi_{t,s}^c + \beta \frac{\partial V_s^B(m_{t+1}^G, m_{t+1}^C)}{\partial m_{t+1}^c} \quad \leq \quad 0, \tag{2}$$

$$-\phi_{t,s}^c + \beta \frac{\partial V_s^S(\tilde{m}_{t+1}^G, \tilde{m}_{t+1}^C, e_{t+1})}{\partial \tilde{m}_{t+1}^c} \quad \leq \quad 0. \tag{3}$$

As is standard in the literature, optimal portfolio choice is independent of the portfolio at the beginning of the CM, so the money distribution is degenerate within each agent type.

Bargaining

When a buyer with portfolio (m^G, m^C) and a seller with portfolio $(\tilde{m}^G, \tilde{m}^C)$ meet they bargain over the triple $(q, \hat{d}^G, \hat{d}^C)$ where q is the quantity of DM-goods produced by the seller and $\hat{d}^c \in \mathbb{R}^+$ is the nominal payment of currency c from the buyer to the seller.Since agents care only about the real value of currency (i.e., value in CM goods), and since this value is unknown at the time of trade (given $s = 0$), we define *expected real balances* as $z_{t,s}^c \equiv m_t^c \, \mathbb{E}_{s'|s}\big[\phi_t^c(s')\big]$ and the *expected real payment* as $d_{t,s}^c \equiv \hat{d}_t^c \, \mathbb{E}_{s'|s}\big[\phi_t^c(s')\big]$. It is therefore convenient to define bargaining over (q, d^G, d^C) directly.

To determine (q, d^G, d^C), we adopt the proportional bargaining solution of Kalai (1977), under which the buyer receives a share $\theta \in [0, 1]$ of the total surplus $\mathcal{S}(q)$ and the seller retains the remainder. Using the linearity of CM value functions with respect to money holdings, we can write the bargaining problem as follows:

$$\max_{q, d^G, d^C} \mathcal{S} \quad \text{s.t.} \quad \theta\Big[-c(q) + d^G + d^C\Big] = (1-\theta)\Big[u(q) - d_t^G - d^C\Big],$$
$$0 \leq d^G \leq z^G, \quad 0 \leq d^C \leq e \cdot z^C.$$

The first constraint is the proportional bargaining condition, while the last two are feasibility constraints ensuring that payments cannot exceed buyers' currency holdings and that crypto transfers are only feasible when sellers can accept them ($e = 1$).

The solution to the bargaining problem is then given by:

$$q_\theta = \begin{cases} q^* & \text{if } z \geq z_\theta(q^*), \\ z_\theta^{-1}(z) & \text{if } z < z_\theta(q^*), \end{cases} \tag{4}$$

where $z_\theta(q) \equiv \theta c(q) + (1-\theta)u(q)$ and $z \equiv z^G + e_t \cdot z^C$. The latter denotes the total stock of *liquid assets* in that match, where the liquidity of crypto is contingent on sellers' ability to recognize it: fully liquid when recognized ($e = 1$) and illiquid otherwise ($e = 0$).

Solution (4) admits the following interpretation: When buyers hold sufficient liquid assets, they purchase the quantity q^*. In this case, if sellers accept both currencies ($e = 1$), the composition of the payment portfolio (d^C, d^G) is indeterminate—any combination of crypto and government currency with total purchasing power $z(q^*)$ is optimal. When q^* is unattainable, however, buyers spend all liquid assets.

By (4), the total trade surplus depends only on the bargaining parameter θ and the expected real value of the buyer's liquid asset portfolio z, so we define $\mathcal{S}(z) \equiv u\big(q_\theta(z)\big) - c\big(q_\theta(z)\big)$. Implicit in this notation is that the surplus depends on whether the seller accepts crypto, since this determines the effective size of the liquid asset portfolio. Due to quasi-linearity, the surplus is independent of the composition of the seller's currency portfolio.

Decentralized Market

Let $\delta_{t,s} \in [0,1]$ be the share of sellers who accept crypto at date t and state s. A buyer entering the DM with real currency portfolio (z^G, z^C) has the following value function:

$$\begin{aligned} V_s^B(z^G, z^C) =& \delta\Big[\max_{q,d^G,d^C} u(q) + \mathbb{E}_{s'|s} W_{s'}^B(z^G - d^G, z^C - d^C)\Big] \\ &+ (1-\delta)\Big[\max_{q,d^G} u(q) + \mathbb{E}_{s'|s} W_{s'}^B(z^G - d^G, z^C)\Big], \end{aligned} \tag{5}$$

subject to the same constraints as in the bargaining problem. A buyer can be matched with a seller who accepts crypto (first line) or a seller who does not (second line). Using quasi-linearity of the CM value function (1), the definition of real balances and the bargaining solutions (4), we can simplify the expression in (5) to:

$$V_s^B(z^G, z^C) = \theta\Big[\delta \mathcal{S}(z^G + z^C) + (1-\delta)\mathcal{S}(z^G)\Big] + z^C + z^G + \mathbb{E}_{s'|s} W_{s'}^B(0,0). \tag{6}$$

Similarly, the value function of a seller with portfolio $(\tilde{z}^G, \tilde{z}^C)$ is:

$$V_s^S(\tilde{z}^G, \tilde{z}^C, e) = (1-\theta)\mathcal{S}(z^G + e \cdot z^C) + \tilde{z}^G + \tilde{z}^C + \mathbb{E}_{s'|s} W_{s'}^S(0,0,e).$$

Currency Demand

Next, we solve for optimal portfolios. Buyers' currency demand follows from their FOCs (2)-(3) combined with (6):

$$\frac{\pi^G_{t+1,s}}{\beta\mathbb{E}_{s'|s}\Omega^G_{t+1,s'}} \geq 1+\theta\Big[(1-\delta_{t+1})\frac{\partial\mathcal{S}(z^G_{t+1,s})}{\partial z^G_{t+1,s}}+\delta_{t+1}\frac{\partial\mathcal{S}(z^G_{t+1,s}+z^C_{t+1,s})}{\partial z^G_{t+1,s}}\Big], \tag{7}$$

$$\frac{\pi^C_{t+1,s}}{\beta\mathbb{E}_{s'|s}\Omega^C_{t+1,s'}} \geq 1+\theta\delta_{t+1}\frac{\partial\mathcal{S}(z^G_{t+1,s}+z^C_{t+1,s})}{\partial z^C_{t+1,s}}, \tag{8}$$

where the corresponding FOC holds with equality whenever $z^c_{t+1,s}>0$ and where:

$$\Omega^c_{t,0}\equiv\frac{\phi^c_{t,1}}{\phi^c_{t,0}},\quad \Omega^c_{t,1}\equiv 1,\quad \pi^c_{t+1,s}\equiv\frac{\phi^c_{t,s}}{\phi^c_{t+1,s}},$$

denote respectively the *currency appreciation factor* and the *gross inflation rate of currency c in state s*. The appreciation factor $\Omega^c_t>1$ ($\Omega^c_t<1$) captures surprise appreciation (depreciation) upon arrival of the sunspot. The inflation rate $\pi^c_{t+1,s}>1$ captures depreciation of currency c within state s (determined by money growth γ^c; see below). We further define *expected gross inflation* as

$$\pi^{c,e}_{t+1,s}\equiv\phi^c_{t,s}/\mathbb{E}_{s'|s}\phi^c_{t+1,s'}=\pi^c_{t+1,s}/\mathbb{E}_{s'|s}\Omega^c_{t+1,s'}$$

where $\pi^{c,e}_{t+1,0}=\pi^c_{t+1,0}/\Big(\rho\Omega^c_{t+1}+(1-\rho)\Big)$ and $\pi^{c,e}_{t+1,1}=\pi^c_{t+1,1}$.

Intuitively, the condition shows that a buyer holds currency c only if its expected inflation cost $\pi^{c,e}_{t+1,s}>\beta$ (left-hand side) is offset by a sufficiently high liquidity premium (right-hand side). The latter is positive whenever some sellers accept currency c in the DM, and is decreasing in currency holdings, reaching zero once total liquid assets are sufficiently large. Absent any liquidity premium, buyers hold the currency only if it yields a sufficiently high real return, i.e., $\pi^{c,e}_{t,s}=\beta$, which corresponds to deflation.

Similarly, we can write the sellers' real demand for currency $c\in\{C,G\}$ as:

$$\pi^{c,e}_{t,s}\geq\beta\ \ (=\ \text{if}\ \tilde{z}^c_{t+1,s}>0). \tag{9}$$

Comparing (7) and (8) with (9), it follows that sellers do not demand a currency with a positive liquidity premium. Moreover, if a currency carries no liquidity premium, the seller is indifferent over holdings of that currency. Going forward, we will therefore assume that sellers never hold any cash, $\tilde{z}^G_{t,s}=0$ for all t and s. However, we still allow sellers to hold crypto currency.

Market Clearing

The market for both currencies clears if $M_t^G = m_{t,s}^G$ and $M_t^C = m_{t,s}^C + \tilde{m}_{t,s}^C$ for all t and s. We can write these conditions in real terms, defining $Z_{t,s}^c \equiv \phi_{t,s}^c M_t^c$ as the *aggregate real value* of currency c where

$$Z_{t,0}^c = \phi_{t,0}^c \left(m_{t,0}^c + \tilde{m}_{t,0}^c\right) = \phi_{t,0}^c \frac{\left(z_{t,0}^c + \tilde{z}_{t,0}^c\right)}{\mathbb{E}_{s'|s=0}\phi_t^c(s')} = \frac{z_{t,0}^c + \tilde{z}_{t,0}^c}{(1-\rho)+\rho\Omega_t^c}, \tag{10}$$

$$Z_{t,1}^c = \phi_{t,1}^c \left(m_{t,1}^c + \tilde{m}_{t,1}^c\right) = z_{t,1}^c + \tilde{z}_{t,1}^c. \tag{11}$$

Equilibrium

Next, we define a particular stationary equilibrium.

Definition 1 (A Crypto-Take-Over Equilibrium)**.** *A crypto-take-over equilibrium is a list of values for all $s \in \{0,1\}$ of expected real balances of buyers (z_s^G, z_s^C), expected real crypto balances of sellers $\tilde{z}_s^C$, aggregate real balances, $\left(Z_s^G, Z_s^C\right)$, consumption q_s, payments (d_s^G, d_s^C) and the share of sellers holding crypto technology δ_s such that*

1. *government currency is only valued before the sunspot, i.e., $Z_0^G > 0$ and $Z_1^G = 0$,*
2. *crypto currency is valued before and after the sunspot, i.e., $Z_0^C > 0$ and $Z_1^C > 0$,*
3. *no seller adopts the crypto technology before the sunspot, i.e., $\delta_0 = 0$, and all sellers adopt it after the sunspot, i.e., $\delta_1 = 1$.*
4. *agents behave optimally, and*
5. *markets for both currencies clear,*

for given preferences $(\beta, u(\cdot), c(\cdot))$, monetary policies (γ^G, γ^C), bargaining power θ, adoption cost χ, and sunspot probability ρ.

While monetary models with multiple intrinsically useless currencies admit many equilibria (Kareken and Wallace (1981)), we are particularly interested in the equilibrium where crypto is valued but not used as a medium of exchange, as observed in real-world crypto markets. Precisely, we seek existence conditions for the equilibrium in which crypto is adopted as a medium of exchange only *after* the sunspot occurs, and analyze its implications. We moreover restrict attention to (a) stationary equilibria and (b) equilibria in which crypto becomes the *sole* medium of exchange once the sunspot occurs — implying that cash loses all value at that point — and (c) all sellers adopt crypto. These restrictions simplify the exposition but are not crucial for understanding the mechanism; we relax (b) and (c) in Section 3.

To solve for this equilibrium, we proceed by backward induction, solving first for the post-sunspot allocation and then for the pre-sunspot allocation. Note that, both pre- and post-sunspot, by stationarity, expected inflation equals the money growth rate conditional on the currency having positive value: $\pi^c_{t,s} = \gamma^c$ whenever $\phi_{t,s} > 0$, for all t, s, and c.

Post-Sunspot

By Definition 1, post-sunspot all agents adopt crypto, which has positive value while cash has none, i.e., $\delta_1 = 1$, $Z^C_1 > 0$ and $Z^G_1 = 0$.[11] Crypto is then the only medium of exchange and the buyer's FOC (7) pins down q_1:

$$\frac{\gamma^C}{\beta} = \frac{u'(q_1)}{[\theta c'(q_1) + (1-\theta)u'(q_1)]} \equiv \mathcal{L}(q), \tag{12}$$

where $\mathcal{L}(q)$ is the *liquidity premium* which satisfies $\mathcal{L}(q) > 0$ for all $q < q^*$, is strictly decreasing in q, and $\mathcal{L}(q^*) = 0$. Therefore, $q_1 = \mathcal{L}^{-1}(\gamma^C/\beta)$ which is strictly decreasing in γ^C/β. In particular, $q_1 > 0$ if and only if

$$\gamma^C \leq \beta/(1-\theta) \equiv \bar{\gamma}^C. \tag{13}$$

Intuitively, higher crypto inflation (or lower β) makes it more costly to carry crypto as a medium of exchange, implying lower DM consumption; for sufficiently high inflation $\bar{\gamma}^C$, buyers prefer not to hold any crypto at all, even at the cost of consuming nothing in the post-sunspot DM.[12] Therefore, $q_1 < q^*$ since $\gamma^C > \beta$, and buyers spend all their crypto holdings so that $z^C_1 = z_\theta(q_1)$. Finally, by (9), sellers hold no crypto due to the liquidity premium $\mathcal{L}(q_1) > 0$ implying that aggregate real balances of crypto currencies are given by $Z^C_1 = z_\theta(q_1) = z_\theta(\mathcal{L}^{-1}(\gamma^C/\beta))$.

A crypto-take-over equilibrium can only exist if two conditions are satisfied: crypto inflation

[11] As is well established, there always exists an equilibrium in which fiat money has no value: anticipating worthless money, agents demand none, which is self-fulfilling.

[12] More precisely, even when crypto maintains a constant value ($\gamma^C = 1$), DM consumption falls to zero for sufficiently low buyer bargaining power θ. This is a particular feature of Kalai bargaining, not shared by Nash bargaining where the marginal benefit of carrying money is always below the cost of holding crypto at that level of consumption.

is not too high, i.e., $\gamma^C \leq \bar{\gamma}^C$ and sellers optimally adopt technology post-sunspot,

$$\beta(1-\theta)\Big(u(q_1) - c(q_1)\Big) \geq \chi,$$

$$\Leftrightarrow \bar{\chi}(\gamma^C) \equiv \beta(1-\theta)\mathcal{S}\Big(z_\theta(\mathcal{L}^{-1}(\gamma^C/\beta))\Big) \geq \chi \tag{14}$$

for q_1 given by (12). The left-hand side of (14) is the benefit of accepting crypto which, given that cash is no longer used, equals the seller's surplus from trade. Observe that a better return on crypto (lower γ^C) increases the incentive to adopt.

Pre-Sunspot

By Definition 1, both cash and crypto have positive value pre-sunspot while no seller has yet adopted crypto: $Z_0^G > 0$, $Z_0^C > 0$, and $\delta_0 = 0$. Because a sunspot occurs with probability ρ and wipes out the value of cash, the pre-sunspot expected return on cash is:

$$\frac{\beta\Big((1-\rho)\phi^G_{t+1,0} + \rho\phi^G_{t+1,1}\Big)}{\phi^G_{t,0}} = \frac{\beta}{\gamma^G}\Big((1-\rho) + \rho\Omega^G\Big) = \frac{\beta}{\gamma^G}(1-\rho),$$

having used the fact that $\phi^G_{t+1,1} = \Omega^G = 0$. Moreover, since only buyers hold cash, by assumption, it follows that $z_0^G = (1-\rho)Z_0^G$. By the buyer's FOC (7), one can pin down q_0:

$$\frac{\gamma^G}{\beta(1-\rho)} = \mathcal{L}(q_0) \tag{15}$$

Therefore, $q_0 = \mathcal{L}^{-1}\Big(\gamma^G/\beta(1-\rho)\Big)$ and $Z_1^G = z_\theta(q_0)$, which are strictly decreasing in γ^G and ρ, and implies $q_0 > 0$ and $Z_0^G > 0$ only when

$$\gamma^G < \beta(1-\rho)/(1-\theta) \equiv \bar{\gamma}^G. \tag{16}$$

The determination of q_0 follows the same logic as for q_1, except that cash inflation — rather than crypto inflation — is the relevant factor, and, more interestingly, higher ρ depresses q_0. Intuitively, since cash loses its value when the sunspot hits, the expected return on cash is decreasing in ρ. Consequently, even when both currencies share the same growth rate ($\gamma^C = \gamma^G$), the mere possibility of a crypto takeover ($\rho > 0$) reduces pre-sunspot consumption ($q_1 > q_0$).

Then, using $\delta_0 = 0$, the buyer's FOC (8) and the seller's FOC (9) coincide. Given $Z_0^C > 0$, the *private option value of crypto* is:

$$Z_0^C = Z_1^C \frac{\beta\rho}{\gamma^C - \beta(1-\rho)} = z_\theta(q_1)\frac{\beta\rho}{\gamma^C - \beta(1-\rho)}. \tag{17}$$

so that $Z_0^C \leq Z_1^C$.[13] Equation (17) makes clear that agents hold crypto pre-sunspot purely as a speculative asset: they anticipate a currency appreciation ($\Omega^C = Z_1^C/Z_0^C > 1$) upon adoption when the sunspot hits, which occurs with probability ρ; with probability $(1-\rho)$, however, crypto depreciates at rate γ^C. The pre-sunspot value Z_0^C adjusts to deliver the required expected return $1/\beta$ so that agents are indifferent to hold crypto. This implies that a lower γ^C and a higher adoption probability ρ both raise the option value Z_0^C.

Since both buyers and sellers are indifferent over how much crypto to hold, the model determines the aggregate value Z_0^C but not the individual portfolios $(z_0^C, \tilde{z}_0^C)$ separately. Denote $\epsilon \equiv z_0^C/(z_0^C + \tilde{z}_0^C)$ as the share of aggregate crypto holdings carried by buyers.

Finally, the existence of a crypto-takeover equilibrium requires two additional conditions: (a) cash inflation is not excessive, $\gamma^G < \bar{\gamma}^G$; and (b) sellers optimally do not adopt crypto before the sunspot occurs. The latter requires:

$$\beta(1-\theta)\Big(\mathcal{S}(z_0^G + z_0^C) - \mathcal{S}(z_0^G)\Big) \leq \chi,$$

$$\Leftrightarrow \underline{\chi}(\gamma^G, \gamma^C, \epsilon) \equiv \beta(1-\theta)\left\{\mathcal{S}\left(z_\theta(q_0) + \epsilon\frac{\rho\gamma^C}{\gamma^C - \beta(1-\rho)} z_\theta(q_1)\right) - \mathcal{S}\left(z_\theta(q_0)\right)\right\} \leq \chi. \quad (18)$$

In words, condition (18) ensures that the pre-sunspot marginal benefit of adoption — given by the seller's share of the incremental trade surplus from crypto adoption, $\mathcal{S}(z_0^G + z_0^C) - \mathcal{S}(z_0^G)$ — does not exceed the adoption cost χ; otherwise, a seller would find it profitable to deviate and adopt already before the sunspot. In particular, $\underline{\chi}(\cdot)$ increases with ρ and falls with γ^C: a better "crypto monetary policy" and a higher probability of it coming into effect both raise the private option value of crypto, increasing the incentive to adopt before the sunspot.

Moreover note that the share of crypto held by buyers, ϵ, affects the incentives of sellers to adopt the crypto technology. As (18) shows, when sellers hold a large fraction of crypto before the sunspot ($\epsilon \to 0$), the incentive of a seller to deviate and adopt crypto is not strong because the seller knows that few buyers are holding crypto so that being able to accept crypto is not worth it. To the contrary, if buyers are the only ones holding crypto, then there are larger gains for sellers from adopting the technology, and the adoption cost should be large to refrain sellers from adopting it.

[13] This pricing formula is only well-defined if $\gamma^C - \beta(1-\rho) > 0$—a condition that always holds given our assumption that $\gamma^C \geq 1$.

To summarize, a crypto-takeover equilibrium consists of a share ϵ, consumption (q_0, q_1), and a private option value Z_0^C solving (12), (15), and (17) for given parameters $(\beta, \theta, \rho, \chi, \gamma^C, \gamma^G)$. Such an equilibrium exists if and only if (13), (14), (16), and (18) are satisfied, i.e., $\gamma^G < \bar{\gamma}^G$, $\gamma^C < \bar{\gamma}^C$, and $\underline{\chi}(\gamma^G, \gamma^C, \epsilon) \leq \chi \leq \bar{\chi}(\gamma^G)$.[14] Sufficient conditions for existence are met whenever γ^C, γ^G, and χ are sufficiently small, θ is not too small, and $\epsilon = 0$ — in which case $\underline{\chi}(\cdot) = 0$.[15] Finally, while ϵ is indeterminate, (q_0, q_1) and Z_0^C are uniquely determined.

Somewhat counterintuitively, the likelihood of a crypto take-over equilibrium is greater when cash is a better means of payment (lower γ^G). As its growth rate falls, cash becomes less expensive to hold, and conditions (16) and (18) are more readily satisfied. The latter follows because as γ^G is low, the trade surplus $\mathcal{S}(\cdot)$ is already close to being maximized. As a result, the ability to accept cryptocurrency pre-sunspot is of little benefit to sellers—and in fact converges to zero as monetary policy approaches the Friedman Rule $\gamma^G = \beta$.

Finally, whether lower γ^C makes existence more likely is ambiguous. On the one hand, it makes holding and adopting crypto post-sunspot more attractive, i.e., (13) and (14) are more likely to be satisfied. On the other, it raises the value of crypto pre-sunspot, thereby increasing the incentive to adopt pre-sunspot.[16]

Discussion

This admittedly stylized framework has many insights. To begin with, the model offers a novel and tractable theory for why an intrinsically useless asset—here, a cryptocurrency—can be valued even in environments where it is not actively used as a medium of exchange. The key mechanism is expectations: agents hold cryptocurrencies not because they are useful today, but because they anticipate they will be useful in the future. This gives holding agents a gain when crypto turn useful. In this sense, cryptocurrencies are not valued because they are a *current* medium

[14]It is important to emphasize that these conditions on χ arise only because money demand is microfounded in our framework. Alternative approaches — such as cash-in-advance or money-in-the-utility-function models — would not deliver them. Although such models may match observed money demand functions well, they are silent on the unobserved demand for a currency not yet used as a medium of exchange.

[15]It is also evident from (18) that $\epsilon = 0$ is not necessary for equilibrium existence; nevertheless, a crypto-takeover equilibrium is more likely to exist for smaller values of ϵ.

[16]It is important to emphasize that even when no crypto-takeover equilibrium exists — perhaps because the incentives to adopt pre-sunspot are too strong — this does not preclude the possibility of an equilibrium in which crypto serves as a medium of exchange both before and after the sunspot.

of exchange, but rather because they are a potential *future* medium of exchange. This is what we call *the private option value of money.*

The model further shows that such an option value does not *generally* exist; it hinges critically on the cost of adoption. If adoption is prohibitively costly, no one adopts it, and the asset is never used. If the cost is too low, everyone adopts immediately, and we return to a standard monetary equilibrium. For the equilibrium to support a speculative valuation without immediate usage, adoption must be costly—but not too costly.[17] This intermediate range creates a gap that supports equilibria in which agents are willing to hold cryptocurrencies in anticipation of future use, but where adoption remains deferred.

Interestingly, the mere possibility of future adoption has real effects today. If agents assign positive probability to the event that crypto will eventually replace cash, then the expected return to holding cash falls. This increases the opportunity cost of liquidity services provided by cash and depresses current consumption. Hence, even when crypto is not yet used in transactions, its anticipated future role can have negative welfare implications in the present. Still, crypto adoption can increase welfare when the growth rate of crypto is low (and lower than the one of cash).

Naturally, the simplicity of the model comes with limitations. While we refer to "cash" and "crypto," the framework is sufficiently abstract that we could as well have labeled them "currency A" and "currency B," or even "Dollar" and "Peso." As such, the model does not yet incorporate key institutional differences—for example, the discretionary nature of fiat supply versus the algorithmic or fixed-supply rules of cryptocurrencies. Incorporating these dimensions, along with other key institutional differences, is a natural extension for a more careful analysis of the normative aspects of the option value of crypto and to bring the model to data, which we do next. We also endogenize γ^G to better understand the role of such "dormant" currency in general, and to address the policy question of how governments respond to the presence of apparently idle cryptocurrencies. Put simply: does society benefit from the existence of crypto and how large is the *social option value of crypto*?

[17] This holds conditional on some buyers holding crypto, i.e., $\epsilon > 0$. If $\epsilon = 0$, the cost of adoption may be zero. Although $\epsilon = 0$ can be an equilibrium in the above exposition, it would be straightforward to modify the model in unproblematic ways to rule out this outcome.

3 The Model With Optimizing Government

We enrich the simple model with a *government good* which can only be provided by the government. All agents derive the following utility from consuming government goods $g_t \in \mathbb{R}^+$ in the CM,

$$v(g_t) = \begin{cases} G \text{ if } g_t \geq \bar{g}, \\ 0 \text{ if } g_t < \bar{g}. \end{cases}$$

Therefore, there is a minimum level of government goods $\bar{g} \in \mathbb{R}^+$, such as basic infrastructure, that the government must provide before agents enjoy government services.[18] Besides providing the public good g_t, the government can also consume CM goods for itself, denoted by $\tilde{x}_t \in \mathbb{R}_0^+$, which benefits neither buyers nor sellers (e.g., vanity projects). The government cannot tax but finances expenditures by printing money at the growth rate $\gamma_t^G \in [1, \infty]$, which it controls.[19] We further assume that G is sufficiently large and $\bar{g}$ sufficiently small such that: (a) $2G - \bar{g} > u(q^*) - c(q^*)$, ensuring that providing the public good is always optimal; and (b) $\bar{g}$ can always be financed through seigniorage. The government budget constraint can then be written as:

$$g_t + \tilde{x}_t + \tau_t^G = \phi_t^G [M_{t+1}^G - M_t^G], \tag{19}$$

where τ_t^G denotes any real transfers made to buyers.

In contrast, the growth rate of crypto is exogenous and fixed at $\gamma^C \geq 0$; any newly issued crypto is lump-sum transferred to buyers, denoted τ^C in real terms.

We replace the sunspot by the exogenous and publicly known event that the benevolent government ($s = g$) becomes self-interested, or rogue ($s = b$). A benevolent government maximizes the utilitarian welfare measure $\mathcal{W}_t$—defined formally below—whereas a rogue government maximizes its own consumption of CM goods, $\tilde{x}_t$. We assume that in $t = 0$ the government is benevolent. The government becomes rogue with probability $\rho \in [0, 1)$ and stays rogue forever. Both government types—taking as given the type of equilibrium (see below)[20]— maximize their

[18] Imposing the step function is not necessary but simplifies the analysis in uncontroversial ways.

[19] Alternatively, one could assume that the government can tax but only up to a limit, with the remainder financed through seigniorage. What matters is that *some* seigniorage revenue is either required or optimal for the government to raise.

[20] As noted by Bassetto (2004), there is some ambiguity in Walrasian models with multiple equilibria and a large player such as the government regarding equilibrium selection. One view has the government move first, potentially forcing a particular equilibrium—e.g., "killing" crypto. However, even if the government eliminates one

objective subject to budget constraint (19), committing to policies $\mathcal{P}_{t,s} \equiv (\gamma^G_{t,s}, \tau^G_{t,s}, g_{t,s})$ once and for all—so that $\mathcal{P}_{t,s} = \mathcal{P}_s$ for all t (without loss of generality given our focus on stationary equilibria), with $\mathcal{P}_0 \neq \mathcal{P}_1$.

Additionally, we model transaction costs that arise with crypto, both for realism and because they eliminate the indeterminacy in the choice of medium of exchange in the bargaining problem. We assume that buyers must pay a real cost $\varphi(d^C_t)$ in the (following) CM for transferring d^C_t (expected, real) units of crypto to the seller in the DM.[21] Specifically, buyers lock $\varphi(x)$ (expected, real) units of cryptocurrency when transferring x units to the seller, where $\varphi(0) = 0$, $\varphi'(0) = 0$, $\varphi'(d^C_t) > 0$ and $\varphi''(d^C_t) > 0$.

Finally, in contrast to the simple model, and to help us match the demand for money in Section 4, we assume that (a) buyers meet a random seller with probability $\sigma \in (0, 1]$ and (b) buyers and sellers receive non-linear utility $U(x)$ from consuming the CM-good x where $U'(x) > 0$, $U''(x) < 0$, $\lim_{x\to 0} U'(x) = \infty$ and $\lim_{x\to\infty} U'(x) = 0$. As in Lagos and Wright (2005), this implies that $x_t = x^*$ for all t and all agents where $U'(x^*) = 1$.

Bargaining

The bargaining problem is as in the simple model, except that buyers' value functions are adjusted for the transaction costs $\varphi(\cdot)$ incurred when paying with crypto. In particular, the adjusted bargaining problem now reads:

$$\mathcal{S}(z^G, e \cdot z^C) \equiv \max_{q, d^C, d^G} \quad u(q) - c(q) - \varphi(d^C)$$
$$\text{s.t.} \quad z_\theta(q) = d^G + d^C + (1-\theta)\varphi(d^C),$$
$$0 \leq d^C + \varphi(d^C) \leq e \cdot z^C, \quad d^G \leq z^C.$$

where, as in the previous section, $z_\theta(q) \equiv \theta c(q) + (1-\theta)u(q)$ and all variables are expressed in expected future real values. Let $D(d^C) \equiv d^C + \varphi(d^C)$ denote the total crypto balance required to deliver d^C crypto units to the seller implying $D(d^C) > d^C$ for all $d^C > 0$. We further define

equilibrium, it remains unclear which of the remaining ones (autarky, cash-only, etc.) the Walrasian auctioneer would select to ensure existence. We therefore adopt the cleaner alternative: the Walrasian auctioneer moves first and selects the equilibrium, and the government takes it as given.

[21] The cost function $\varphi(d^C_t)$ can be taken as a primitive. Alternatively, it can be interpreted as the outcome of bargaining between the buyer and a miner who facilitates transactions, faces a convex cost, and bears all exchange-rate risk on $\phi^C_{t,s}/\phi^G_{t,s}$.

$\bar{d}^C(q, d^G)$ as the crypto payment d^C that satisfies the proportional bargaining condition for a given (q, d^G); formally $\bar{d}^C$ solves $\bar{d}^C - \theta\varphi(\bar{d}^C) = z_\theta(q) - d^G$.

Lemma 1. *The solution to the bargaining problem is given by:*

$$q = \begin{cases} q^* & \text{if } z_\theta(q^*) \leq z^G, \\ \bar{q}(z^G) & \text{if } z_\theta(q^*) > z^G \text{ and } \bar{d}^C + \varphi(\bar{d}^C) < e \cdot z^C, \\ z_\theta^{-1}\Big(z^G + e \cdot z^C - \theta\varphi(D^{-1}(e \cdot z^C))\Big) & \text{if } z_\theta(q^*) > z^G \text{ and } \bar{d}^C + \varphi(\bar{d}^C) > e \cdot z^C, \end{cases}$$

where $\bar{d}^C = \bar{d}^C(q, z^G)$ *and* $\bar{q}(z^G)$ *is the unique solution to:*[22]

$$\frac{u'(q) - c'(q)}{(1-\theta)u'(q) + \theta c'(q)} = \frac{\varphi'\Big(\bar{d}^C(q, z^G)\Big)}{1 + (1-\theta)\varphi'\Big(\bar{d}^C(q, z^G)\Big)}.$$

The proof can be found in Appendix D.1. The transaction cost $\varphi(d^C)$ introduces a pecking order among means of payment as Figure 1 illustrates. When buyers hold sufficient cash balances ($z^G \geq z_\theta(q^*)$), they purchase the first-best quantity q^* using cash alone (Region 1). Only when cash is scarce ($z^G < z_\theta(q^*)$) do buyers resort to crypto. How much crypto buyers spend depends on the marginal transaction cost, which in turn depends on their cash holdings. Conditional on $z^G < z_\theta(q^*)$, optimal consumption falls to $\bar{q}(z^G) < q^*$. Attaining $\bar{q}(z^G)$ requires sufficient crypto balances; if buyers hold enough, they supplement their cash with some crypto (Region 2). Otherwise, buyers exhaust both currencies (Region 3). As we show below, only the latter case arises in equilibrium; however, characterizing existence requires the full set of bargaining outcomes, since a deviating seller who matches with a crypto-holding buyer may trade off-path in Region 2.

[22] In terms of payments, we obtain:

$$\Big(d^G, d^C\Big) = \begin{cases} \Big(z_\theta(q^*), 0\Big) & \text{if } z_\theta(q^*) \leq z^G, \\ \Big(z^G, D^{-1}(\bar{d}^C)\Big) & \text{if } z_\theta(q^*) > z^G \text{ and } \bar{d}^C + \varphi(\bar{d}^C) < e \cdot z^C, \\ \Big(z^G, D^{-1}(z^C)\Big) & \text{if } z_\theta(q^*) > z^G \text{ and } \bar{d}^C + \varphi(\bar{d}^C) > e \cdot z^C. \end{cases}$$

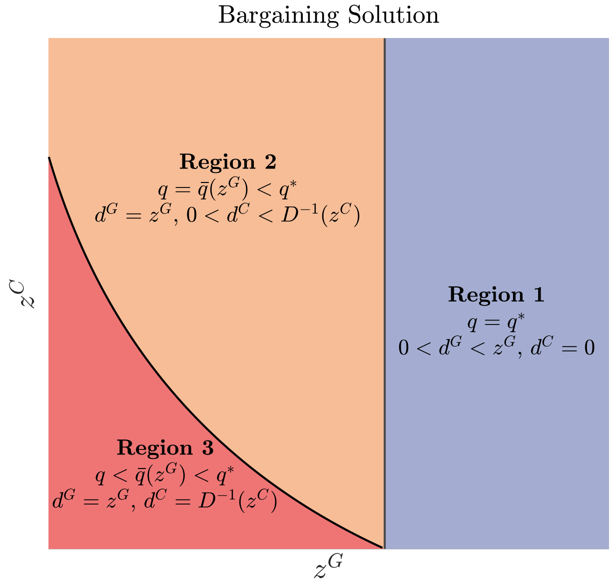


Figure 1: Bargaining solution for the case $e = 1$.

Currency Demand

Currency demand is almost identical to that in Section 2, but adjusted for the matching probability σ and surplus $\mathcal{S}(z^G, z^C)$ (including the transaction cost $\varphi(\cdot)$):

$$\frac{\pi^{G,e}_{t+1,s}}{\beta} \geq 1 + \sigma\theta \left[\delta \frac{\partial \mathcal{S}(z^G_t, z^C_t)}{\partial z^G_t} + (1-\delta)\frac{\partial \mathcal{S}(z^G_t, 0)}{\partial z^G_t}\right], \tag{20}$$

$$\frac{\pi^{C,e}_{t+1,s}}{\beta} \geq 1 + \sigma\theta\delta \frac{\partial \mathcal{S}(z^G_t, z^C_t)}{\partial z^C_t}, \tag{21}$$

where, as in Section 2, expected inflation is given by $\pi^{c,e}_{t+1,0} \equiv \pi^c_{t+1,0} \Big/ \Big(\rho\Omega^c_{t+1} + (1-\rho)\Big)$ and $\pi^{c,e}_{t+1,1} \equiv \pi^c_{t+1,1}$.

Adoption

The seller's optimal adoption choice follows the same logic as in Section 2: adopting is strictly beneficial (detrimental) if and only if $\chi > (<)\, \bar{\chi}(s)$, where

$$\bar{\chi}(s) \equiv \sigma(1-\theta)\Big[\mathcal{S}(z^G_s, z^C_s) - \mathcal{S}(z^G_s, 0)\Big]; \tag{22}$$

if $\chi = \bar{\chi}(s)$, sellers are indifferent and may or may not adopt; in particular, only a fraction $\delta \in (0,1)$ may do so.

Welfare

To set a goal for the benevolent government, and to make normative statements, we assume a social welfare function that treats buyers, sellers, and the rogue government[23] equally (each with measure 1):

$$\mathcal{W}_t \equiv V^B_{t,0}(z^G_t, z^C_t) + V^S_{t,0}(\tilde{z}^G_t, \tilde{z}^C_t) + V^G_{t,0},$$

where the rogue government's continuation value is $V^G_{t,0} = \mathbb{E}_{s'|s=0} \sum_{j=t}^{\infty} \beta^j \tilde{x}_{j,s'} \cdot \mathbb{I}_{s'}$, where $\mathbb{I}_{s'}$ is an indicator for whether the government is rogue, with $\mathbb{I}_1 = 1$ and $\mathbb{I}_0 = 0$. Inserting the value functions yields:

$$\mathcal{W}_t = \mathbb{E}_{s'|s=0} \sum_{j=t}^{\infty} \beta^j \Big[\sigma \delta_{j,s'} \mathcal{S}(z^G_{j,s'}, z^C_{j,s'}) + \sigma(1-\delta_{j,s'})\mathcal{S}(z^G_{j,s'}, 0) - \delta_{j,s'} \cdot \chi + 2v(g_{j,s'}) - g_{j,s'} + \tilde{x}_{j,s'}\Big] + \frac{2\left(U(x^*) - x^*\right)}{1-\beta}.$$

Observe that—due to quasi-linearity— (γ^c, τ^c) do not appear directly in the welfare function, as they cancel in the aggregate.[24]

Government Policy

The rogue and the benevolent governments both commit to a policy $\mathcal{P}_s \equiv (\gamma^G_s, \tau^G_s, g_s)$ for each regime s. That policy maximizes the government's objective conditional on the prevailing equilibrium—in particular, the government takes the adoption choice δ_s as given.

The policy of the rogue government that comes to power at date $t = \tilde{t}$ solves:

$$\mathcal{P}^*_1 = \arg\max_{\gamma^G} \sum_{t=\tilde{t}}^{\infty} \beta^{t-\tilde{t}} \tilde{x}_{t,1}(\mathcal{P}_1) = \arg\max_{\gamma^G_1} \sum_{t=\tilde{t}}^{\infty} \beta^{t-\tilde{t}} Z^G_{t,1}(\gamma^G_1) \cdot (\gamma^G_1 - 1),$$

where in the last expression we have inserted the government budget constraint (19) and used $g^*_1 = \tau^{G,*} = 0$—hence, the rogue government maximizes seigniorage revenue.

The benevolent government, by assumption in power at $t = 0$ and taking $\mathcal{P}^*_1$ as given, solves

$$\mathcal{P}^*_0 = \arg\max_{\mathcal{P}_0} \mathcal{W}_0(\mathcal{P}_0, \mathcal{P}^*_1).$$

Note that, in a stationary environment, $\tilde{x}^*_{t,0} = 0$ for all $t < \hat{t}$ and $\gamma^{G,*}_0 > 1$. The latter follows the assumption that financing government services $\bar{g}$ is always beneficial.

[23] We could also include the benevolent government's utility, but since the benevolent government values $\mathcal{W}_t$, this would simply scale $\mathcal{W}_t$ up by a factor.

[24] See Molico (2006) for a search-theoretic monetary model without quasi-linearity.

Market Clearing

Market clearing is still given by (10) and (11).

Equilibrium Definition

Next we define a stationary equilibrium. Let us denote the date at which the government goes rogue simply as "the event."

Definition 2 (Stationary Monetary Equilibrium)**.** *A stationary monetary equilibrium is a list of currency portfolios* $\{z_s^c, \tilde{z}_s^c\}_{c,s}$*, aggregate real balances,* $\{Z_s^c\}_{c,s}$*, DM-quantities* $\{q_s\}_s$*, adoption shares* $\{\delta_s\}_s$*, inflation rates* $\{\pi_s^c\}_{s,c}$*, government policy* $\{\mathcal{P}_s\}_s$ *and currency appreciation* Ω_s^c *for all* $s \in \{0, 1\}$*,* $c \in \{G, C\}$ *and* $j = \{B, S\}$ *such that*

1. *cash is always valued, i.e.,* $Z_0^G > 0$ *and* $Z_1^G > 0$*,*
2. *no seller adopts the crypto technology before the event, i.e.,* $\delta_0 = 0$*,*
3. *the government chooses its policy* $\mathcal{P}_s$ *to maximize its objective,*
4. *buyers and sellers behave optimally, and*
5. *the markets for both currencies clear,*

for given buyer preferences $(\beta, u(\cdot), U(\cdot))$*, seller preferences* $c(\cdot)$*, government preferences* $(\bar{g}, \rho)$*, crypto technology* $(\gamma^C, \varphi(\cdot), \chi)$*, bargaining power* θ*, and matching probability* σ*.*

Our analysis restricts attention to monetary equilibria satisfying three conditions: (a) crypto is never adopted before the government turns rogue; (b) cash is always valued; and (c) real balances are constant conditional on state s. Condition (a) selects (given $\delta_1 > 0$; see below) precisely the kind of equilibrium we seek to study: one with option value on crypto. Condition (b) excludes cases where agents coordinate to abandon cash entirely once the government goes rogue—a scenario we regard as less empirically relevant; we relegate these *crypto-takeover* equilibria to Appendix B. Finally, condition (c) is for simplicity and implies that $\pi_{t,s} = \gamma_{t,s}$ conditional on currency c having value.

Moreover, any monetary equilibrium in which all sellers adopt the technology after the event ($\delta_1 = 1$) is called a *full-adoption equilibrium*; if only some adopt ($1 > \delta_1 > 0$), it is called a *partial-adoption equilibrium*; and any monetary equilibrium with $\delta_1 = 0$ is called a *no-adoption equilibrium*. In the main text we focus on full-adoption and no-adoption equilibria, i.e., $\delta_1 \in \{0, 1\}$; the analysis of the partial-adoption equilibrium is discussed in Appendix C.

3.1 No-Adoption Equilibrium

We first characterize the benchmark in which cryptocurrency is never adopted post-event ($\delta_1 = 0$). We proceed backward and first characterize the equilibrium outcome post-event. To write the solution to the buyer's problem it is useful to define the liquidity premium

$$\mathcal{L}(q) \equiv \sigma\theta \, \frac{u'(q) - c'(q)}{\theta c'(q) + (1-\theta)u'(q)},$$

which is a decreasing function of q and is zero at the efficient allocation q^*.

Post-Event

With $\delta_1 = 0$, crypto has no value post-event, i.e, $Z_1^C = 0$, and buyers only pay with cash. Then, given cash inflation as set by the rogue government, γ_1^G, the buyer's FOC pins down DM output $q_1(\gamma_1^G)$ as the unique solution to

$$\frac{\gamma_1^G}{\beta} = 1 + \mathcal{L}(q). \tag{23}$$

Output q_1 is strictly decreasing in γ_1^G and falls to zero at a finite threshold $\bar{\gamma}^G$. The value of cash is then $Z_1^G(\gamma_1^G) = z_\theta\Big(q_1(\gamma_1^G)\Big)$.

The rogue government maximizes seigniorage revenue $R_1(\gamma^G) \equiv z_\theta(q_1(\gamma^G))(\gamma^G - 1)$ choosing γ^G. Denoting the solution by $\gamma_1^{G,*}(0)$—where "(0)" stands for no-adoption—we necessarily have $\gamma_1^{G,*}(0) < \bar{\gamma}^G$ because cash has no value when the government chooses $\gamma_1^{G,*}(0)$ above that threshold.

Pre-Event

Since crypto is not adopted post-event, it has no value then, and as it is not used pre-event ($\delta_0 = 0$), it commands no value pre-event either , i.e., $Z_0^C = 0$.

Given γ_0^G as set by the government pre-event and expecting $\gamma_1^G (= \gamma_1^{G,*}(0))$ post-event, the DM output $q_0(\gamma_0^G, \gamma_1^G)$ is given by the first order condition (20),

$$\frac{\gamma_0^G}{\beta\big[(1-\rho) + \rho\,\Omega^G(q_0, \gamma_1^G)\big]} = 1 + \mathcal{L}(q_0), \tag{24}$$

where the currency appreciation factor on cash upon the event is

$$\Omega^G = \Omega^G(q_0, \gamma_1^G) \equiv (1-\rho)\,\frac{\dfrac{z_\theta(q_1(\gamma_1^G))}{z_\theta(q_0)}}{1-\rho\,\dfrac{z_\theta(q_1(\gamma_1^G))}{z_\theta(q_0)}}.$$

As we show in Appendix D.2, there always exists a unique $q_0(\gamma_0^G, \gamma_1^G)$ solving (24) where $q_0(\gamma_0^G, \gamma_1^G) < q_1(\gamma_1^G)$ if and only if $\gamma_0^G < \gamma_1^G$. In words, (a) higher current inflation γ_0^G always depresses q_0, and (b) the mere prospect of the government turning rogue ($\gamma_1^G > \gamma_0^G$) already reduces output today by lowering expected currency appreciation Ω^G, thereby raising the cost of holding cash. The latter, in turn, mitigates the impact of current inflation: reducing the real value of cash today—while holding the value of future cash constant—causes currency appreciation to increase.[25] See Figure 2 for a visualization.

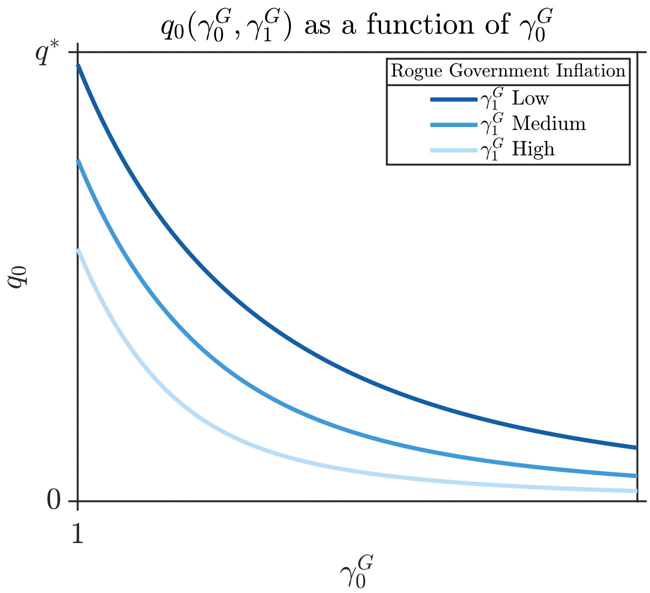


Figure 2: Pre-event consumption q_0 as a function of current and future cash inflation (γ_0^G, γ_1^G)

Finally, since γ_0^G reduces output and thereby lowers welfare, the benevolent government will optimally choose the lowest γ_0^G that generates just enough seigniorage revenue to finance $\bar{g}$.

[25] A corollary is that there is no upper bound on γ_0^G beyond which trade collapses: $q_0 > 0$ even as $\gamma_0^G \to \infty$. This contrasts with the standard result under Kalai bargaining, where sufficiently high inflation drives q_0 to zero, and follows from the countervailing effect via currency appreciation Ω^G.

The existence and uniqueness of this equilibrium, together with that of the full-adoption equilibrium, are established jointly in Proposition 2 below.

This baseline makes clear that once the government turns rogue, it finances its consumption through an inflation tax, which—as an unintended consequence—depresses both future output q_1 and, through lower expected currency appreciation, current output q_0. Welfare is therefore lower in a world with a potentially rogue government ($\rho > 0$) than in one without ($\rho = 0$).

3.2 Full-Adoption Equilibrium

Post-Event

With $\delta_1 = 1$, buyers consider holding both currencies in their portfolio. Both currencies carry the same liquidity premium, but crypto additionally bears a transaction cost discount since each payment incurs a fee $\varphi(d^C)$. Therefore, to write the solution to the buyers' problem it is useful to define the *transaction cost discount*

$$\mathcal{T}(q_1, z_1^C) \equiv \sigma\theta \frac{u'(q_1)}{\theta c'(q_1) + (1-\theta)u'(q_1)} \cdot \frac{\varphi'\big(D^{-1}(z_1^C)\big)}{1 + \varphi'\big(D^{-1}(z_1^C)\big)}.$$

Taking γ_1^G and γ_1^C as given, the buyer's FOCs become

$$\frac{\gamma_1^G}{\beta} \geq 1 + \mathcal{L}(q_1), \qquad \frac{\gamma^C}{\beta} \geq 1 + \mathcal{L}(q_1) - \mathcal{T}(q_1, z_1^C), \tag{25}$$

holding with equality whenever the respective currency is held in positive amounts. Since $\gamma_1^c > \beta$ for both currencies c, the seller's FOCs imply that sellers never hold any currency post-event as they have no demand for liquidity.

The spread in inflation rates $\gamma_1^G - \gamma_1^C$ governs the currency mix in the buyers' portfolio. In Appendix D.3, we formally show that (a) buyers use only cash when $\gamma_1^G \leq \gamma_1^C$, (b) buyers use only crypto when $\gamma_1^G \geq \hat{\gamma} < \bar{\gamma}$ for some threshold $\hat{\gamma}$ (defined in the Appendix), and (c) buyers use both currencies when $\gamma_0^C < \gamma_0^G < \hat{\gamma}$. In the latter case, the spread equals the transaction cost discount, i.e.,:

$$\gamma_1^G - \gamma_1^C = \mathcal{T}(q_1, z_1^C).$$

The rogue government maximizes seigniorage, $R(\gamma^G) = Z_1^G(1)(\gamma^G - 1)$, and we denote its choice of inflation by $\gamma_1^{G,1}(1)$, where "(1)" denotes crypto adoption post event.

Post-Event Equilibrium

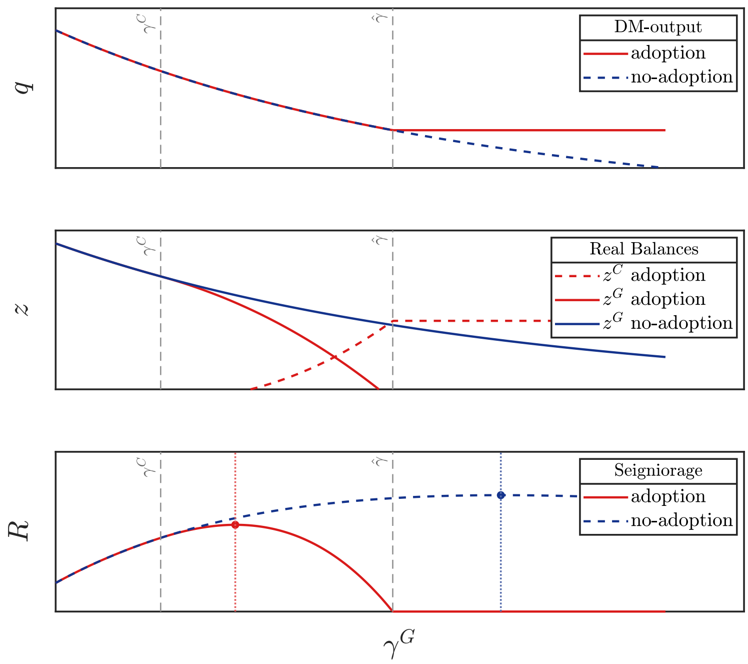


Figure 3

Figure 3 illustrates currency demand and (seigniorage) revenues across the full-adoption and no-adoption equilibria. The top panel shows DM output q_1: in both equilibria, a higher γ_1^G reduces output for low values of γ_1^G. Under full adoption, however, q_1 becomes independent of γ_1^G for all $\gamma_1^G \geq \hat{\gamma}$—at that point, no agent wishes to hold cash. This is reflected in the middle panel, which displays aggregate real balances: Z_1^G declines more rapidly under full adoption because crypto ($Z_1^C > 0$, dot-dashed) absorbs part of liquidity demand, driving it to zero at $\hat{\gamma}$, whereas under no adoption Z_1^G remains strictly positive for all $\gamma_1^G < \bar{\gamma}^G$ and $Z_1^C = 0$ throughout. The bottom panel translates these effects into the rogue government's objective: the introduction of crypto compresses both the level and the slope of the seigniorage curve, shifting the revenue-maximising inflation rate from $\gamma_1^{G,*}(0)$ down to $\gamma_1^{G,*}(1)$. As the following result establishes, this *disciplining effect* of crypto on the rogue government holds in general:

Proposition 1 (Disciplining Effect)**.** *If* $\gamma^C < \gamma_1^{G,*}(0)$*, then*

$$\gamma_1^{G,*}(1) \in \left[\gamma^C,\, \min\{\hat{\gamma},\, \gamma_1^{G,*}(0)\}\right).$$

The proof can be found in Appendix D.5. When all sellers accept crypto post-event, the rogue government is compelled to set a strictly lower inflation rate on cash than it would in the no-adoption equilibrium. The intuition is straightforward: buyers can substitute into crypto whenever cash inflation rises, and at $\gamma_1^G \geq \hat{\gamma}$ they do so entirely, leaving no seigniorage base. Anticipating this, the rogue government optimally restrains inflation. As a by-product, the lower inflation raises output q_1.

Pre-Event

Given the inflation rate post-event $\gamma_1^G (= \gamma_1^{G,*}(1))$, and setting $\delta_0 = 0$ in the buyer's FOC, the pre-event value of crypto satisfies

$$Z_0^C = \rho \, \frac{\beta \, Z_1^C(\gamma^C, \gamma_1^G)}{\gamma^C - \beta(1-\rho)}. \tag{26}$$

Agents hold crypto not for its current liquidity—no seller accepts it yet—but because of the currency appreciation they expect to realise once adoption occurs. This is again the private option value of crypto.

Furthermore, pre-event output q_0 is purchased with cash only and remains determined by (24). It therefore depends on the currency appreciation factor Ω on cash. Whether q_0 is higher or lower under full adoption relative to the no-adoption benchmark hinges on the relative strength of two opposing forces acting on the post-event difference in cash value, $Z_1^G(1) - Z_1^G(0)$, and thereby on the currency appreciation factor differential $\Omega(1) - \Omega(0)$:

- **Disciplining effect on q_0.** The lower post-event inflation $\gamma_1^{G,*}(1) < \gamma_1^{G,*}(0)$ raises total currency demand, $z_\theta\Big(q_1(\gamma_1^{G,*}(1))\Big) > z_\theta\Big(q_1(\gamma_1^{G,*}(0))\Big)$, and thereby increases the post-event value of cash, amplifying the currency appreciation factor Ω. This raises the expected return to holding cash pre-event and hence stimulates q_0.

- **Replacement effect.** The presence of crypto diverts part of post-event liquidity demand away from cash, replacing it with crypto. Consequently, for a given (γ_1^G, q_1), the real value of cash is lower under $\delta_1 = 1$ than under $\delta_1 = 0$. This lowers the expected return to holding cash pre-event and hence depresses q_0.

If the disciplining effect dominates, q_0 is higher under full adoption and crypto raises pre-event welfare. If the replacement effect dominates, q_0 is lower and the mere *prospect* of future crypto adoption depresses the current value of cash and, in turn, pre-event consumption. Which force prevails is an empirical question addressed in Section 4.

Finally, as before, the benevolent government sets the lowest inflation rate γ_0^G consistent with financing $\bar{g}$. Somewhat paradoxically, even though inflation always rises once the government turns rogue in the no-adoption equilibrium, cash-inflation may actually *fall* under full adoption whenever the disciplining effect is sufficiently strong.[26]

3.3 Existence and Uniqueness

We now characterize the conditions under which the full-adoption and no-adoption equilibria exist and, in particular, whether they can coexist.

Proposition 2. *Suppose $\theta > \bar{\theta} \equiv (1-\beta)/(\beta\sigma)$, which is necessary for any monetary equilibrium to exist.*

(i) ***No-adoption equilibrium.*** *The equilibrium with no crypto adoption post-event ($\delta_1 = 0$) exists for all $\chi \geq 0$. The equilibrium with no adoption is unique.*

(ii) ***Full-adoption equilibrium.*** *The equilibrium with full crypto adoption post-event ($\delta_1 = 1$) exists for all $\bar{\chi}(1) \geq \chi \geq 0$ (see equation (22)). The equilibrium with full adoption is unique when buyers do not hold crypto pre-event $\epsilon = 0$.*

Notice that both equilibria coexist whenever $\chi \leq \bar{\chi}(1)$. When $\chi > \bar{\chi}(1)$, however, only the no-adoption equilibrium exists. When this inequality fails, adoption costs are too high to sustain any equilibrium belief about future adoption, eliminating any scope for the disciplining and replacement effects. Consequently, only when $\chi \leq \bar{\chi}$ does coordination on the crypto equilibrium constitute a credible threat to the rogue government. Finally, note that $\bar{\chi}(1) > 0$ requires $\theta < 1$: sellers bear an adoption cost only if they stand to gain from it.

[26] For example, suppose that $\varphi(d^C) \approx 0$ for all d^C sufficiently small and $\gamma^C = 1$. In that case, the post-event government is forced to set $\gamma_1^G \approx 1$. The benevolent government, however, may need to set $\gamma_0^G > 1$ to finance $\bar{g}$ pre-event, and it can do so since it does not face competition from crypto. This is not unlike what we find in Section 4.

The Societal Option Value of Money

The previous sections have shown that the allocation differs markedly depending on whether the economy is in a no-adoption equilibrium ($\delta = 0$)—either because no viable alternative currency exists or because private agents do not perceive it as such—or in a full-adoption equilibrium ($\delta = 1$), in which crypto is valued for its option value. To assess whether the presence of a dormant secondary currency with option value is socially beneficial, we compare the welfare implications across these two equilibria. The welfare function, expressed as a function of adoption $\delta \in \{0, 1\}$ and consumption compensation $\Delta \in \mathbb{R}^+$, is given by

$$
\begin{aligned}
\mathcal{W}(\delta, \Delta) = & \frac{\sigma}{1-\beta(1-\rho)} \mathcal{S}_0(\delta, \Delta) + \frac{\rho\beta\sigma}{\big(1-\beta(1-\rho)\big)(1-\beta)} \mathcal{S}_1(\delta, \Delta) \\
& + \frac{2}{1-\beta}\big(U(x^*\Delta) - x^*\big) + \frac{1-\rho}{1-\beta(1-\rho)}\big(2v(g^*) - g^*\big) \\
& - \frac{\rho}{\big(1-\beta(1-\rho)\big)(1-\beta)} \delta \cdot \chi,
\end{aligned}
\tag{27}
$$

where $\mathcal{S}_0(\delta, \Delta) \equiv u(q_0(\delta)\Delta) - c(q_0(\delta))$ and $\mathcal{S}_1(\delta, \Delta) \equiv u(q_1(\delta)\Delta) - c(q_1(\delta)) - \delta\varphi(d_1(\delta))$ denote equilibrium DM surplus in states 0 and 1, respectively. Observe that adoption δ only affects (a) pre-event DM surplus $\mathcal{S}_0$, (b) post-event DM surplus $\mathcal{S}_1$, and (c) the direct adoption cost χ. The CM good and government good allocations x^* and g^* are determined independently of δ and hence drop out of $\mathcal{W}(1, 1) - \mathcal{W}(0, \Delta^*)$. See Appendix D.7 for details on the derivation of (27).

We define the *societal option value of money* as the consumption compensation Δ^* solving $\mathcal{W}(1, 1) = \mathcal{W}(0, \Delta^*)$. That is, $\Delta^* - 1$ measures the permanent proportional increase in total consumption (both DM and CM goods) that would make an agent indifferent between the no-adoption and the full-adoption equilibrium.

4 Quantitative Analysis

Data and Calibration

In order to quantify the model—and especially, to estimate the social option value of money—, we calibrate it to the US economy, using Bitcoin (BTC) as the representative cryptocurrency. As argued above, the latter fits the description of potential future money with a fixed supply schedule, which a rogue government would find difficult to ban given its decentralized design.

We adopt the following assumptions on preferences and technology:

$$\begin{aligned} u(q) &= \frac{(q+b)^{1-\nu} - b^{1-\nu}}{1-\nu}, \\ c(q) &= q, \\ U(x) &= B \cdot \log(x), \\ \varphi(d) &= \exp\Big(C_1 + C_2 \log(d)\Big), \end{aligned}$$

where $\nu \in \mathbb{R}^+$ is the *relative risk aversion* and $B \in \mathbb{R}^+$ can be interpreted as the *importance of centralized exchange*. We set $b \approx 0$ such that $u(0) = 0$ and $\lim_{q \to 0} u'(q) \approx \infty$, and G sufficiently large so that the benevolent government always sets $\gamma_0^G > 1$ to finance $\bar{g}$.

There are 11 parameters to calibrate: the discount factor β, the coefficient of relative risk aversion ν, the matching probability σ, the bargaining power of buyers θ, the importance of centralized exchange B, the transaction cost parameters (C_1, C_2), the cost of adoption χ, the threshold government spending $\bar{g}$ (which determines $\gamma_1^{G,*}$), the probability that the government turns rogue ρ, and the growth rate of crypto γ^C.

We calibrate the model at a quarterly frequency and employ a multi-step calibration strategy. In the first step, we fix three parameters exogenously: (a) the discount factor $\beta = 1/(1+r) = 0.99$, implying a quarterly real interest rate of 1%, consistent with Lagos and Wright (2005); (b) the growth rate of the crypto supply to zero, i.e., $\gamma^C = 1$, given that Bitcoin's supply is currently around 20 million and is hardcoded to cap at 21 million by around 2140, implying an average annual growth rate of just 0.063%—negligible for our purposes; and (c) $\bar{g}$ such that the optimal monetary growth rate $\gamma_0^{G,*}$ corresponds to an quarterly (annual) inflation rate of $\approx 0.86\%$ (3.5%).

In the second step, we calibrate $\Gamma \equiv (B, \nu, \sigma, \theta)$ to a version of the model without crypto (i.e., $\rho = 0$), following Lucas (2000) and, more specifically, Lagos and Wright (2005). Using data from 1951–2008—the year of Bitcoin's introduction—we set these parameters to match the empirical money demand function and aggregate markups (see below).

In the third step, we calibrate (C_1, C_2) to match the fee-payment relationship (see below).

In the fourth step, we set ρ to match the relative real balances between dollars and Bitcoin (reflecting BTC's exchange value), conditional on $(\beta, \gamma^C, \bar{g}, \Gamma, C_1, C_2)$ (see below).

Finally, since a necessary condition for crypto to have positive value is $\chi \in [0, \bar{\chi}]$, we remain agnostic about the precise value of χ and instead report welfare bounds corresponding to the lower and upper bounds of this interval.

This multi-step approach is preferable to estimating everything together for two reasons. First, it facilitates comparison of our estimates with the literature—especially regarding the cost of inflation, which is closely related to the social option value of money. Second, and more importantly, the "deep parameters" $(\beta, \bar{g}, \Gamma)$ are well-identified from long-run features of US money demand that predate Bitcoin and are harder to discipline once the crypto block is introduced. Separating the two steps thus ensures that the estimation of the deep parameters is not contaminated by the relatively noisy Bitcoin-specific moments. The key assumption is that the deep parameters have not changed markedly over the 2008–2026 period, which we regard as a reasonable approximation given the stability of long-run money demand relationships.[27]

Calibration Γ

In the first step set Γ to match long-run money demand and average markups.

The money demand function is a relationship between the opportunity cost of holding money and a measure of aggregate money holdings. Money demand (per unit of output) is given by $L \equiv M/(PY)$, where M is a monetary aggregate, P is the price level, and Y is aggregate output. We measure the opportunity cost of holding money, i, by the federal funds rate, nominal GDP as a measure of PY, and M by the "New M1" constructed by Lucas Jr and Nicolini (2015).[28]

We construct L in the model by noting that aggregate nominal output is the sum of output in the centralized market, $x^*/\phi^G = B/\phi^G$, and the decentralized market, $\sigma \cdot z_\theta(q_0)/\phi^G$ (see equation (23)), so that $PY = \big(B + \sigma \cdot z_\theta(q_0)\big)/\phi^G$. Money holdings are $M = z_\theta(q_0)/\phi^G$, and the opportunity cost is $i = (\gamma^G/\beta - 1)/\sigma$. The model-implied money demand is therefore:

$$L(i, \Gamma) = \frac{z_\theta\Big(q(i, \Gamma)\Big)}{B + \sigma \cdot z_\theta\Big(q(i, \Gamma)\Big)},$$

where $(\hat{L}_t, \hat{i}_t)$ denotes the empirical money demand and opportunity cost in quarter t.

Average markups, denoted by $\hat{\mu}$, are defined as the ratio of prices to marginal cost, averaged across industries. Given the ongoing debate about the level and trend of markups in the U.S. economy, we consider a range of values. We follow Basu and Fernald (1997) for a conservative

[27] See, for instance, Benati et al. (2021) for international evidence on the long-run stability of money demand.

[28] This series accounts for money market accounts, which after regulatory changes post-1980 became equally liquid as checking accounts, yielding a stable long-run relationship between the opportunity cost of holding money and the demand for near-monies.

lower bound of $\hat{\mu} = 1.1$ and De Loecker et al. (2020) for an upper bound of $\hat{\mu} = 1.6$, corresponding to their late-sample estimate. While the latter has been contested on methodological grounds (Traina, 2018), we include it to ensure our results are robust to the possibility of substantially higher markups.

In the model, markups differ between the two markets: (a) in the decentralized market, marginal cost equals 1 while prices are (implicitly) given by $z_\theta(q)/q$, implying a markup whenever $\theta < 1$; and (b) in the centralized market, goods are traded at marginal cost and hence carry no markup. The model-implied aggregate markup is then given by:

$$\mu(i, \Gamma) = \Big(1 - \alpha(i, \Gamma)\Big) \frac{z_\theta\Big(q(i, \Gamma)\Big)}{q(i, \Gamma)} + \alpha(i, \Gamma),$$

where $\alpha(i, \Gamma) \equiv B/\Big(B + z_\theta(q(i, \Gamma))\Big)$ is the expenditure share of the centralized market.

We then set Γ so as to minimize the following:

$$\min_{\Gamma} \Big\{ \Big[\sum_{t=0}^{T} \Big(L(\hat{i}_t, \Gamma) - \hat{L}_t \Big) \Big]^2 + \Big[\mu(\Gamma, \bar{i}) - \hat{\mu} \Big]^2 \Big\},$$

where $\bar{i} = \sum_{t=0}^{T} i_t / N$ denotes the empirical average interest rate. Figure 4a shows that the model does a good job of targeting money demand. The model similarly matches aggregate markups well.

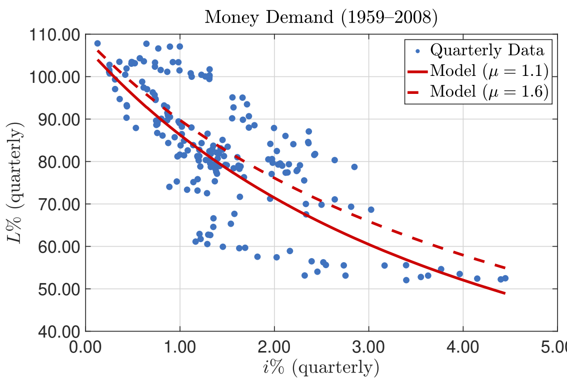
Calibration (C_1, C_2)

Recall that φ denotes the real transaction cost as a function of real Bitcoin transactions d, with the relationship governed by (C_1, C_2).[29]

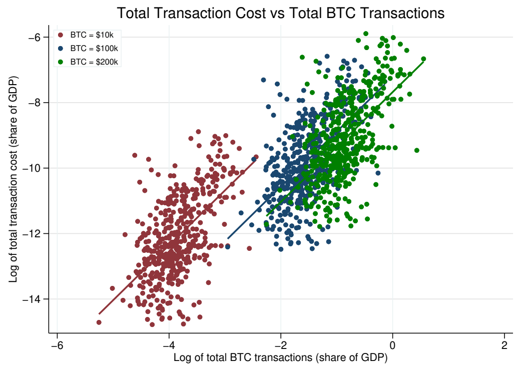
To estimate (C_1, C_2), we use two weekly series from `blockchain.com`: total transaction fees $\hat{\varphi}_t$ (in BTC per quarter) and total Bitcoin transactions $\hat{d}_t$ (in BTC per quarter).[30] Since the model requires a relationship between φ and d expressed in CM goods rather than in BTC, one cannot regress $\hat{\varphi}_t$ on $\hat{d}_t$ directly.

[29]In the model, d^C is the real *expected* value of Bitcoin transactions. Since we do not directly observe this expected value, we use realized Bitcoin values instead. The discrepancy between the two is negligible, as Bitcoin's price moves little over the very short duration of a single transaction.

[30]The former is the total BTC value of all transaction fees paid to miners per day, aggregated to weekly frequency, while the latter is the total estimated value of transactions on the blockchain per day, in BTC, aggregated to weekly frequency.



(a) US money demand



(b) Bitcoin transaction cost

Figure 4: The left panel (a) shows empirical money demand alongside the model-implied money demand from the calibrated model. The right panel (b) shows the empirical relationship between log BTC transaction volume (as a share of GDP in US dollars) and log BTC transaction cost (as a share of GDP in US dollars), together with the model-implied transaction cost function $\varphi(\cdot)$ for each BTC price assumed.

Instead note that by assumption

$$\log\Big(\underbrace{\hat{\varphi}_t \cdot \phi_t^C}_{\equiv \varphi_t}\Big) = C_1 + C_2 \log\Big(\underbrace{\hat{d}_t \cdot \phi_t^C}_{\equiv d_t}\Big).$$

Subtracting $\log(Y_t) \equiv \log(B + \sigma z_\theta(q))$ from both sides, and expressing both transaction fees and transaction volume as shares of GDP in dollars, this becomes:

$$\log\left(\hat{\varphi}_t \cdot \frac{\phi_t^C}{Y_t}\right) = \log\left(\hat{\varphi}_t \cdot \varepsilon_t \cdot \frac{\phi_t^G}{Y_t}\right) = C_1 + C_2 \log\left(\hat{d}_t \cdot \varepsilon_t \cdot \frac{\phi_t^G}{Y_t}\right) - (1 - C_2)\log(Y_t),$$

where $\varepsilon_t \equiv \phi_t^C / \phi_t^G$ is the price of BTC in dollars. Since ε_t is highly volatile in the data—with fluctuations driven not only by adoption-related or regulatory news but also by other factors—and since changes in Y are likewise unrelated to the transaction-cost margin we seek to identify, we follow a fixed exchange rate approach over the entire sample to prevent short-run noise from contaminating our estimates. We therefore treat both $\varepsilon_t = \varepsilon$ and Y as constant:

$$\log\left(\hat{\varphi}_t \cdot \varepsilon \cdot \frac{\phi^G}{Y}\right) = \underbrace{C_1 + (C_2 - 1)\log(Y)}_{\equiv: \tilde{C}_1} + C_2 \log\left(\hat{d}_t \cdot \varepsilon \cdot \frac{\phi^G}{Y}\right).$$

Using data on US nominal GDP ($\approx$\$22 trillion) and considering three alternative BTC prices, $\varepsilon \in \{\$10\text{k}, \$100\text{k}, \$200\text{k}\}$ (a wide range reflecting the well-documented volatility of cryptocurrency prices), we estimate $(\tilde{C}_1, C_2)$ by OLS, using $\log\left(\hat{\varphi}_t \cdot \varepsilon \cdot \frac{\phi^G}{Y}\right)$ as the dependent variable and $\log\left(\hat{d}_t \cdot \varepsilon \cdot \frac{\phi^G}{Y}\right)$ as the regressor—both of which are directly observable. The structural parameters are then recovered as $C_2 = \tilde{C}_2$ and $C_1 = \tilde{C}_1 - (C_2 - 1)\log(Y)$.[31]

Table 1: Implied Transaction Fees as Share of GDP (%)

Calib.	μ	BTC	Vol = 50% GDP	Vol = 100% GDP
1	1.1	\$10k	0.1109%	0.3545%
2	1.1	\$100k	0.0233%	0.0745%
3	1.1	\$200k	0.0146%	0.0466%
4	1.6	\$10k	0.0908%	0.2903%
5	1.6	\$100k	0.0191%	0.0610%
6	1.6	\$200k	0.0119%	0.0382%

Notes: Each entry reports the implied real transaction cost φ as a percentage of real output under different assumptions for aggregate BTC payment volume and BTC price.

Table 1 shows implied real transaction costs as a share of GDP, for three BTC price levels and two volume scenarios. We find that the real cost of transacting with BTC, even when volume equals 100% of GDP, are fairly small ranging from 0.01% to 0.35% of GDP.

Calibration ρ

We calibrate ρ by matching the empirical ratio of the dollar money supply to the Bitcoin supply (in dollar terms) to its model counterpart, Z_0^G/Z_0^C; this ratio is decreasing in ρ, since Z_0^C is increasing in ρ (see equation (26)). For the Bitcoin supply we use the 2026 figure of $\approx$ 20 million coins, and for the dollar money supply the 2026 M2 aggregate of $\approx$ \$22 trillion. As before, we consider $\varepsilon \in \{\$10\text{k}, \$100\text{k}, \$200\text{k}\}$. The implied annualized probability of a rogue government takeover is reported in Table 2.[32]The estimates range from 0.2% to around 3% per year—a fairly

[31]For Y, the model implies $\log(B + \sigma z_\theta(q))$ evaluated at the average interest rate in our sample. Targeting $\mu \in \{1.1, 1.6\}$ yields $Y = 0.99$ and $Y = 1.33$, respectively.

[32]Bitcoin's price reflects speculation, momentum, store-of-value demand, regulatory arbitrage, and many other factors the model cannot decompose; the implied ρ of 0.2%–3% should therefore not be interpreted as a serious estimate of government rogue risk, but rather as whatever residual makes the model fit Bitcoin's price given all other assumptions. The wide price interval—spanning \$10,000 to \$200,000 (with the all-time high recorded to

low probability, even under the most optimistic BTC price assumption of $\varepsilon = \$200{,}000$.

Results

Table 2 reports the results for each of the six calibrations (see table 3 in Appendix A). The first column shows the implied annualized probability of the government going rogue, inferred from the Bitcoin price ϵ and respective money supplies. These probabilities are fairly small, indicating that — given our theory — the observed value of Bitcoin is consistent with only a modest probability of actual government corruption.

Table 2: Calibration Results

			Government	Full-adoption ($\delta_1 = 1$)			No-adoption ($\delta_1 = 0$)			Option Value
Calib.	μ	BTC	$\rho(\%)$	$\gamma_1^G(\%)$	q_0	q_1	$\gamma_1^G(\%)$	q_0	q_1	$\Delta^*(\%)$
1	1.1	\$10k	0.20%	0.39%	0.6388	0.7589	76.06%	0.6351	0.1041	[0.10%, 0.10%]
2	1.1	\$100k	0.64%	0.27%	0.6353	0.7639	76.06%	0.6238	0.1041	[0.28%, 0.28%]
3	1.1	\$200k	1.48%	0.27%	0.6290	0.7643	76.06%	0.6032	0.1041	[0.56%, 0.55%]
4	1.6	\$10k	0.44%	0.38%	0.5656	0.7067	46.34%	0.5570	0.0689	[0.19%, 0.18%]
5	1.6	\$100k	1.04%	0.27%	0.5626	0.7127	46.34%	0.5427	0.0689	[0.39%, 0.38%]
6	1.6	\$200k	2.96%	0.26%	0.5538	0.7131	46.34%	0.5011	0.0689	[0.81%, 0.79%]

Notes: $\Delta^*(\%) = \Delta^* \cdot 100$, and $\rho(\%)$ and $\gamma_1^G(\%)$ denote the annualized equivalents in percentage, i.e., $\rho(\%) \equiv \rho \cdot 400$ and $\gamma^G(\%) \equiv ((\gamma^G)^4 - 1) \cdot 100$.

The second column reports the implied equilibrium inflation rate of the rogue government (γ_1^G) and the equilibrium DM output both before and after the event (q_0 and q_1), conditional on BTC adoption. The third column reports the same objects for the no-adoption equilibrium.

Three patterns stand out. First, absent adoption, the seigniorage-maximizing inflation rate—46.34% or 76.06% depending on the calibration of μ—is substantial but well within the range of estimates reported in the literature.[33] Second, the seigniorage-maximizing inflation rate is

date at \$126,080)—expresses our agnosticism about Bitcoin's long-run value, while still imposing some discipline on plausible values of ρ.

[33]Easterly et al. (1995) estimate the seigniorage-maximizing inflation rate for a panel of eleven high-inflation developing countries over 1960–1990. Estimates vary considerably across countries and specifications: where a finite Laffer curve maximum exists, it ranges from around 67% (Israel) to 376% (Peru), while a substantial share of country-specification combinations yield no interior maximum, implying that seigniorage increases monotonically with inflation. Our implied values of 46.34% and 76.06% are at the lower end of their estimates. To the extent

substantially lower in the full-adoption equilibrium than in the no-adoption equilibrium, reflecting a strong disciplining effect that curtails the rogue government's ability to inflate. Consequently, full adoption prevents a large output collapse (e.g., from 0.64 to 0.11 in Calibration 1). This disciplining force is powerful because Bitcoin has a fixed supply—guaranteeing long-run price stability—and, as shown above, entails relatively low transaction costs. Third, the fact that pre-event output is also higher under full adoption implies that the disciplining effect is strong enough to dominate the replacement effect, resulting in an expected currency appreciation on cash holdings.

The last column reports the social option value of Bitcoin. Despite a fairly low probability of the government turning rogue, the combination with a powerful disciplining effect yields significant option values, ranging from 0.1% to 0.8% of consumption. To put this in perspective, at an average annual household consumption of $75,000, agents would be willing to pay between $75 and $600 per year to live in a world where Bitcoin exists as an alternative currency.

How do we explain the wide range of these numbers? Note that calibrations targeting higher markups μ and higher BTC prices ε yield larger option values, while the option value is virtually invariant to the cost of adoption χ. The first pattern follows from the fact that higher BTC prices imply a higher probability of the government turning rogue, ρ—raising the probability that the option is exercised. The second follows from the well-established result that the welfare cost of inflation is higher in economies where sellers have greater bargaining power and charge higher markups—as evidenced by the lower q_0—raising the value of the option once exercised.[34] The third follows from an internal model implication: χ must be sufficiently low to ensure that sellers' incentive to adopt crypto is small—since post-event buyers carry both cash and crypto, the marginal benefit of accepting crypto is limited. Hence, the option comes at little direct cost.

To conclude, even though Bitcoin may *seem* to serve no useful social purpose beyond speculation, our model suggests that it provides a substantial option value—akin to the premium one would pay for an option to use an alternative currency and thereby insure society against the risk of governments turning rogue.

that our search-theoretic framework underestimates the true seigniorage-maximizing inflation rate, our estimates of the social option value of money should be interpreted as conservative lower bounds.

[34]Craig and Rocheteau (2008) show that the welfare cost of inflation is substantially higher when sellers have greater bargaining power, since buyers become more sensitive to inflation and reduce money holdings faster—shrinking the tax base and inducing the rogue government to choose a lower inflation rate despite higher markups.

5 Conclusion

We propose a novel theory—the option value of money—that explains why intrinsically useless currencies are valued: not because they currently serve as a medium of exchange, but because they might do so in some future state of the world. We set up a microfounded model of money demand, derive a closed-form pricing formula for cryptocurrencies, and characterize the conditions under which option value arises: in particular, we show that option value can only arise if the adoption cost is neither too high (making adoption not worthwhile) nor too low (prompting immediate adoption).

We then ask whether having a currency with only option value can be socially beneficial. We show that, in theory, there are both positive and negative effects on welfare. On the one hand, the potential of an alternative currency being adopted reduces the demand for the currency that is currently used, raising the cost of holding it and therefore depressing current consumption. On the other hand, the possibility of using cryptocurrencies as a medium of exchange can be an effective hedge against a government that may seek to abuse its monopoly over money creation in the future.

Calibrating our model using US and Bitcoin data, we show that, in spite of a very low market-implied probability of Bitcoin adoption, there are potentially large welfare benefits from having Bitcoin. Households would be willing to forgo between 0.10% and 0.81% of consumption to live in a world where Bitcoin carries option value.

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Appendix A Tables

Table 3: Calibrations

Calib.	μ	BTC	σ	θ	η	B	C_1	ρ	$\bar{\chi}$
1	1.1	\$10k	0.06	0.95	0.65	0.95	−5.64	0.0005	0.0003
2	1.1	\$100k	0.06	0.95	0.65	0.95	−7.20	0.0016	0.0003
3	1.1	\$200k	0.06	0.95	0.65	0.95	−7.67	0.0037	0.0003
4	1.6	\$10k	0.05	0.80	0.74	1.30	−5.65	0.0011	0.0013
5	1.6	\$100k	0.05	0.80	0.74	1.30	−7.40	0.0026	0.0013
6	1.6	\$200k	0.05	0.80	0.74	1.30	−7.87	0.0074	0.0013

Notes: $\beta = 0.99$, $b = 0.0001$ $C_2 = 1.68$, $\gamma^C = 1$ and $\bar{g}$ such that $\gamma_0^G = 1.0086$ in all calibrations.

Appendix B Take-Over Equilibrium

In Section 3, we focused on equilibria where cash has value after the event. Here, we analyze *take-over* equilibria, characterized by $Z_1^C > 0$ and $Z_1^G = 0$, in which crypto fully displaces cash. Since cash has no value in these equilibria, the policy of the government is irrelevant. As before, we distinguish between full adoption, $\delta_1 = 1$, and partial adoption, $\delta_1 \in (0, 1)$. Relative to the analysis in the main text, the only conceptual difference arises in the post-event economy.

Full-Adoption

A full-adoption equilibrium is given by $z_1^C = Z_1^C$ such that:

$$\frac{\gamma^C}{\beta} = 1 + \sigma\theta \frac{\partial \mathcal{S}(0, Z_1^C)}{\partial Z_1^C},$$

$$\chi \leq \sigma(1-\theta)\mathcal{S}(0, Z_1^C) \equiv \bar{\chi}.$$

The first equation pins down Z_1 as a decreasing function of $\gamma^C < \bar{\gamma}^C$; for $\gamma_1^C \geq \bar{\gamma}^C$ buyers hold no crypto. The second equation is a necessary condition for the equilibrium to hold and is satisfied if χ and γ^C are sufficiently low. Denote the value of crypto under full-adoption by $\bar{Z}_1^C$.

Partial-Adoption

A partial-adoption equilibrium is given by $z_1^C = Z_1^C$ and δ_1 such that:

$$\chi = \sigma(1-\theta)\mathcal{S}(0, Z_1^C),$$
$$\frac{\gamma^C}{\beta} = 1 + \sigma\theta\delta_1 \frac{\partial \mathcal{S}(0, Z_1^C)}{\partial Z_1^C}.$$

A solution Z_1^C to the first equation exists if and only if $\chi \leq \sigma(1-\theta)\mathcal{S}(0, \bar{Z}_1^C)$, which is precisely the condition for the full-adoption equilibrium to exist. Hence, the partial-adoption equilibrium can only exist if the full-adoption equilibrium exists. Moreover, it follows that $Z_1^C \leq \bar{Z}_1^C$. Denote the solution to the first equation by $\hat{Z}_1^C$.

The second equation implicitly defines $Z_1^C(\delta_1, \gamma_1^G)$, which is increasing in δ_1 and decreasing in γ_1^G. A partial-adoption equilibrium moreover requires the existence of some $\delta_1 \in (0,1)$ such that $Z_1^C(\delta_1, \gamma_1^G) = \hat{Z}_1^C$. Since $\lim_{\delta_1 \to 0} Z_1^C(\delta_1) = 0$, $\lim_{\delta_1 \to 1} Z_1^C(\delta_1) = \bar{Z}_1^C$, and $\hat{Z}_1^C \leq \bar{Z}_1^C$, the intermediate value theorem guarantees that such a δ_1 exists whenever $\hat{Z}_1^C < \bar{Z}_1^C$, or equivalently $\chi < \bar{\chi}$. Therefore, a partial-adoption equilibrium exists if and only if $\chi < \bar{\chi}$, whereas the full-adoption equilibrium exists if and only if $\chi \leq \bar{\chi}$.

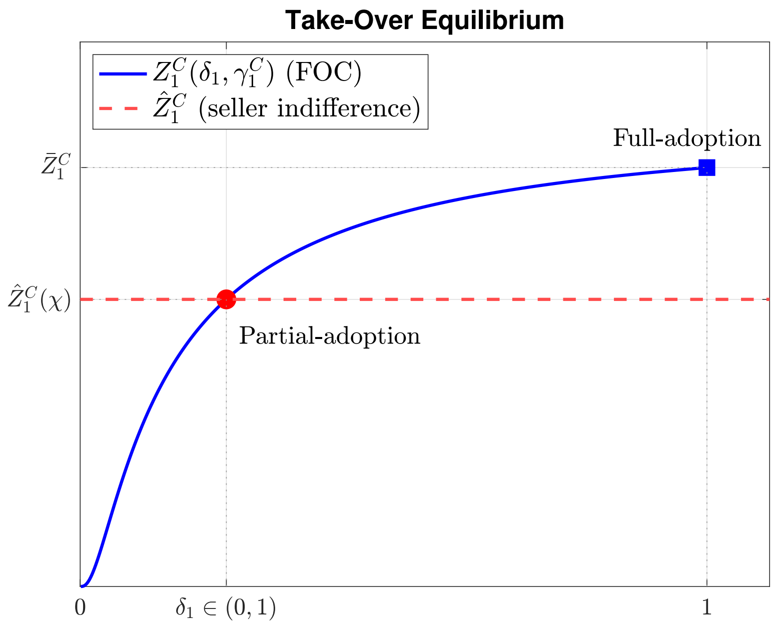

Figure 5: Existence of both full-adoption and partial-adoption equilibria under take-over scenario

Figure 5 illustrates this multiplicity of equilibria: when $\chi < \bar{\chi}$, there exist (a) a full-adoption equilibrium in which all sellers accept crypto, and (b) a partial-adoption equilibrium in which only a fraction

$\delta_1^* \in (0,1)$ do so.[35] The intuition is straightforward: if sellers expect crypto to be of little value, only a subset will adopt, depressing crypto demand and thereby confirming its low value. Conversely, under more optimistic beliefs, full adoption is sustained, supporting correspondingly higher crypto valuations.

Appendix C Partial-Adoption Equilibrium

This section studies the partial-adoption equilibrium conditional on cash retaining value (see Appendix B for the case where cash loses value), focusing on post-event implications; pre-event analysis is conceptually unchanged. Note that $Z_1^C > 0$ throughout by implication. Denote DM output in matches where crypto is accepted by q_1, and in those where it is not by $\tilde{q}_1$.

Post-Event

By assumption, post-event only some sellers adopt crypto, i.e., $\delta_1 \in (0,1)$. This requires that sellers are indifferent between adopting and not-adopting. The post-event equilibrium $(\delta_1, z_1^G, z_1^C, q_1, \tilde{q}_1, d_1^C)$ conditional on γ_1^G then solves:

$$\chi = \sigma(1-\theta)\Big[u(q_1) - c(q_1) - \varphi(d_1^C) - [u(\tilde{q}_1) - c(\tilde{q}_1)]\Big], \tag{C.1}$$

$$\frac{\gamma_1^G}{\beta} = 1 + \delta_1 \mathcal{L}(q_1) + (1-\delta_1)\mathcal{L}(\tilde{q}_1), \tag{C.2}$$

$$\frac{\gamma^C}{\beta} = 1 + \delta_1 \mathcal{L}(q_1) - \mathcal{T}(q_1, z_1^C), \tag{C.3}$$

where $q_1 \equiv z_\theta^{-1}\Big(z_1^G + z_1^C - \theta\varphi(d_1^C)\Big)$, $\tilde{q}_1 \equiv z_\theta^{-1}(z_1^G)$ and $d_1^C = D^{-1}(z_1^C)$. Compared to the full-adoption equilibrium, equation (C.1) is new and ensures seller indifference. As in the full-adoption equilibrium, equations (C.2)–(C.3) are the FOCs from the buyer's portfolio problem.

Note that a lower adoption share δ_1 raises the marginal benefit of holding cash—since $\mathcal{L}(q_1) < \mathcal{L}(\tilde{q}_1)$ and $q_1 > \tilde{q}_1$—while simultaneously reducing the marginal benefit of holding crypto. However, δ_1 is endogenously determined and, as the following proposition shows, adjusts to changes in (γ_1^G, γ^C) in rather non-intuitive ways.

Proposition 3. *Suppose a partial-adoption equilibrium exists.*[36] *Then an increase of cash-inflation reduces adoption of crypto while reducing real balances of both currencies, i.e.,* $\frac{dz_1^C}{d\gamma_1^G} < 0$, $\frac{dz_1^G}{d\gamma_1^G} < 0$ *and* $\frac{d\delta_1}{d\gamma_1^G} < 0$. *Moreover, a higher level of crypto-inflation increases adoption of crypto while reducing real balances of both currencies, i.e.,* $\frac{dz_1^C}{d\gamma^C} < 0$, $\frac{dz_1^G}{d\gamma^C} < 0$ *and* $\frac{d\delta_1}{d\gamma^C} > 0$.

[35] When $\chi = \bar{\chi}$, only the full-adoption equilibrium exists, as the seller-indifference locus coincides with $\bar{Z}_1^C$.

[36] Closed-form existence results are unavailable; however, as shown in Figure 6, these equilibria exist for some parameters.

We relegated the proof to Appendix D.6. As in the full-adoption case, higher inflation on currency c reduces demand for—and hence the value of—currency c. However, in a departure from the full-adoption case, the seller indifference condition (C.1) introduces a novel interdependence: any decrease in z_1^c must be accompanied by a decrease in $z_1^{c'}$, where $c' \neq c$. Intuitively, if crypto becomes more valuable—and hence more attractive for sellers to accept—the value of cash must increase to keep adoption incentives in check.

As a consequence, an increase in cash inflation does not raise the value of crypto as in the full-adoption equilibrium—on the contrary, it *reduces* the value of crypto by lowering δ_1 and hence the liquidity premium of crypto, $\delta_1 \mathcal{L}(q_1)$. The mechanism is as follows: higher cash inflation reduces the demand for cash, and to keep sellers indifferent, the value of crypto must fall. Absent any adjustment in δ_1, crypto becomes relatively more attractive than cash for buyers, so to restore buyer indifference between the two, δ_1 must fall. The intuition for crypto-inflation is analogous.

How does a partial-adoption equilibrium discipline the rogue government? Interestingly, while a full-adoption equilibrium always disciplines the rogue government (Proposition 1), a partial-adoption equilibrium may yield higher rogue inflation than a no-adoption equilibrium. To see why, note that optimal cash-inflation is given by:

$$\gamma^{G,*}(\delta) = 1 + \frac{Z_1^G(\gamma^{G,*} \mid \delta)}{-\frac{\partial Z_1^G(\gamma^{G,*} \mid \delta)}{\partial \gamma^G}}.$$

Note that, $Z_1^G(\gamma^{G,*} \mid 0) > Z_1^G(\gamma^{G,*} \mid \delta)$ for all $\delta > 0$ which is simply the replacement effect. However, in the partial-adoption equilibrium we also have

$$-\frac{\partial Z_1^G(\gamma^{G,*} \mid \delta = 0)}{\partial \gamma^G} > -\frac{\partial Z_1^G(\gamma^{G,*} \mid 0 < \delta < 1)}{\partial \gamma^{G,*}}.$$

That is, under partial adoption, higher cash inflation reduces the value of cash *less* than under no adoption, since—as noted above—higher inflation also reduces δ_1. The government thus has an additional incentive to inflate: namely, to crowd out the competing cryptocurrency. Depending on the relative strength of these two effects, either a disciplining effect or a *reverse disciplining effect* may arise. Figure 6 provides a numerical illustration of the reverse disciplining effect.

Appendix D Proofs

D.1 Proof Lemma 1

Proof. The bargaining problem is given by:

$$\max_{q, d^G, d^C} u(q) - d^G - d^C - \varphi(d^C)$$

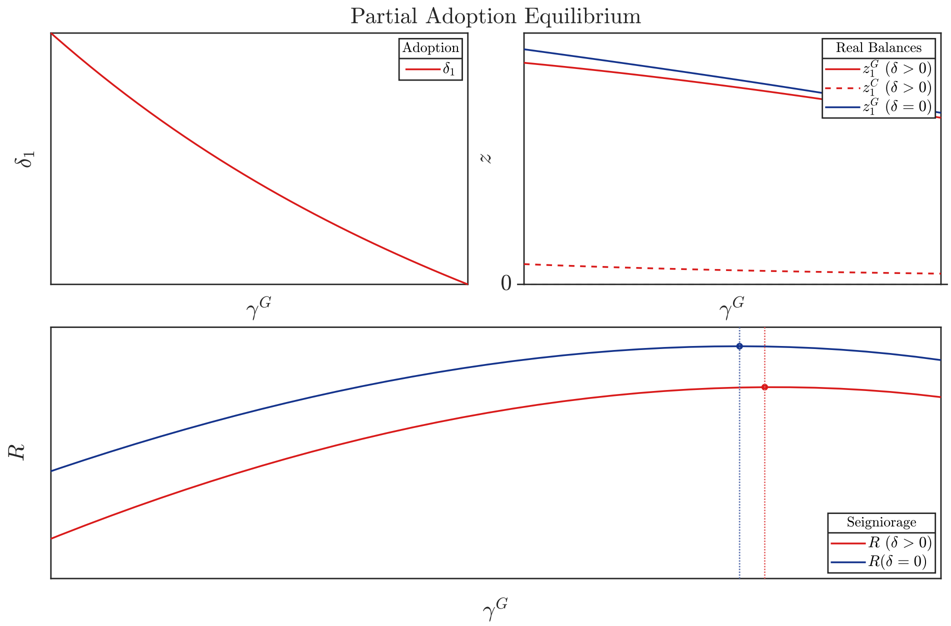

Figure 6: Functional forms are identical to Section 4. Parameters in the example are given by: $\beta = 0.99$, $\theta = 0.3$, $\nu = 0.5$, $C_1 = -0.1$, $C_2 = 1.2$, $\sigma = 0.5$, $\gamma_C = 1$ and $\chi = 0.01$

$$s.t. \;\; d^G + d^C + (1-\theta)\varphi(d^C) = \theta c(q) + (1-\theta)u(q) \equiv z_\theta(q)$$
$$0 \leq d^G \leq z^G,$$
$$0 \leq d^C + \varphi(d^C) \leq e \cdot z^C.$$

The FOCs of this problem with respect to q, d^C and d^G are respectively given by:

$$u'(q) - \lambda[\theta c'(q) + (1-\theta)u'(q)] = 0,^{37} \tag{D.1}$$

$$-1 + \lambda - \mu^G \leq 0, \tag{D.2}$$

$$-1 + \lambda \frac{(1 + (1-\theta)\varphi'(d^C))}{(1 + \varphi'(d^C))} - \mu^C \leq 0, \tag{D.3}$$

where $\lambda > 0$ is the Lagrange multiplier for the proportional bargaining constraint and $\mu^c \geq 0$ is the Lagrange multiplier for the feasibility constraint for currency $c = G, C$.

We need to differentiate four cases:

1. $\underline{\mu^G = \mu^C = 0}$: First, we show that $d^C = 0$. Suppose not. Then by (D.3):

$$\lambda = \frac{(1 + \varphi'(d^C))}{(1 + (1-\theta)\varphi'(d^C))} > 1.$$

[37] We assume that the FOC with respect to q always holds with equality which is trivially true whenever $z^G > 0$ or $e \cdot z^C > 0$. Otherwise, $q = d^C = d^G = 0$.

But this is a contradiction with (D.2) which implies $\lambda \leq 1$. Second, given $d^C = 0$ and $q > 0$ it follows from proportional bargaining constraint that $d^G > 0$. This, in turn, implies that $\lambda = 1$ by (D.2). Using this in (D.1) implies $q = q^*$. By the feasibility constraint for cash then $z^G \geq z_\theta(q^*)$.

2. $\underline{\mu^G = 0 \text{ and } \mu^C > 0}$: The latter immediately implies that

$$\lambda = \frac{(1+\varphi'(d^C))}{(1+(1-\theta)\varphi'(d^C))}[1+\mu^C] > 1.$$

But this is a contradiction with (D.2) implying $\lambda \leq 1$.

3. $\underline{\mu^G > 0 \text{ and } \mu^C = 0}$: First, we show that $d^C > 0$. Suppose not. Then (D.3) implies $\lambda \leq 0$. But this is a contradiction since (D.2) implies $\lambda > 1$. Hence, $d^G = z^G$ and $d^C > 0$. Equation (D.3) implies:

$$\lambda = \frac{(1+\varphi'(d^C))}{(1+(1-\theta)\varphi'(d^C))}.$$

Inserting this condition into (D.1):

$$\frac{u'(q)}{\theta c'(q) + (1-\theta)u'(q)} = \frac{(1+\varphi'(d^C))}{(1+(1-\theta)\varphi'(d^C))}. \tag{D.4}$$

Denote by $\bar{d}(q, z^G)$ the value of d^C solving the proportional bargaining condition for a given (q, z^G), i.e., $z^G + \bar{d}(q, z^G) + (1-\theta)\varphi(\bar{d}(q, z^G)) = z_\theta(q)$ for all q and z^G. Note that $\bar{d}(0, z^G) = 0$, $\bar{d}(q, z_\theta(q)) = 0$, $\partial\bar{d}(q, z^G)/\partial q > 0$ and $\partial\bar{d}(q, z^G)/\partial z^G < 0$ for all q and $z^G < z_\theta(q)$.

Inserting $d^C = \bar{d}(q, z^G)$ in (D.4) yields:

$$LHS(q) \equiv \frac{u'(q)}{\theta c'(q) + (1-\theta)u'(q)} = \frac{1+\varphi'\Big(\bar{d}(q, z^G)\Big)}{1+(1-\theta)\varphi'\Big(\bar{d}(q, z^G)\Big)} \equiv RHS(q).$$

Let us denote the implicit solution of q to the previous equation by the correspondence $\bar{q}^C(z^G)$. Next, we will demonstrate the following properties for any $z^G < z_\theta(q^*)$: First, we show that $\bar{q}^C(z^G)$ always exists, second $\bar{q}^C(z^G)$ is a function, third $\bar{q}^C(z^G) < q^*$, and fourth $\partial\bar{q}^C(z^G)/\partial z^G > 0$:

- **Existence**: Observe that $\lim_{q\to 0} LHS(q) = \infty$, $\lim_{q\to 0} RHS(q) = 1$, $\lim_{q\to q^*} LHS(q) = 1$ and $\lim_{q\to q^*} RHS(q) > 1$ for any $z^G < z_\theta(q^*)$. Therefore, by continuity of both $RHS(q)$ and $LHS(q)$, $\bar{q}^C(z^G)$ exists for all $z^G < z_\theta(q^*)$.
- **Uniqueness**: $\bar{q}^C(z^G)$ is unique since $LHS'(q) < 0$ and $RHS'(q) > 0$.
- **Boundedness**: Directly follows from the facts that a) $LHS(q^*) - RHS(q^*) < 0$ and b) $LHS(q) - RHS(q)$ is a strictly decreasing function.
- **Derivative**: By the implicit function theorem:

$$\partial\bar{q}^C(z^G)/\partial z^G = -\frac{\partial\Big[LHS(q) - RHS(q)\Big]/\partial z^G}{\partial\Big[LHS(q) - RHS(q)\Big]/\partial q}.$$

It is straightforward to ascertain that numerator is positive while denominator is negative. Thus, $q = \bar{q}^C(z^G)$ and $d^C = \bar{d}(\bar{q}^C(z^G), z^G)$ given that $d^C + \varphi(d^C) \leq e \cdot z^C$ or:

$$\bar{d}\Big(\bar{q}^C(z^G), z^G\Big) + \varphi\Big(\bar{d}(\bar{q}^C(z^G), z^G)\Big) \leq e \cdot z^C.$$

4. $\underline{\mu^G > 0 \text{ and } \mu^C > 0}$: In that case, $d^G = z^G$ and $d^C + \varphi(d^C) = e \cdot z^C$ which implies that $d^C = D^{-1}(e \cdot z^C)$ where $D(d^C) \equiv d^C + \varphi(d^C)$. Furthermore, the proportional bargaining constraint then implies:

$$\begin{aligned} q &= z_\theta^{-1}\Big(z^G + d^C + (1-\theta)\varphi(d^C)\Big), \\ &= z_\theta^{-1}\Big(z^G + e \cdot z^C - \theta \cdot \varphi(D^{-1}(e \cdot z^C))\Big), \end{aligned}$$

where the last line follows from the definition of $D(d^C) = d^C + \varphi(d^C)$ and $D(d^C) = e \cdot z^C$.

■

D.2 Proof Lemma 2

Lemma 2. *There always exists a unique* $0 < q_0(\gamma_0^G, \gamma_1^G) < q^*$ *solving* (24) *for all* $1 \leq \gamma_0^G$ *and* $1 \leq \gamma_1^G < \bar{\gamma}^G$*. Moreover,* $q_0(\gamma_0^G, \gamma_1^G)$ *is strictly decreasing in* γ_0^G *and* γ_1^G*.*

Proof. Define

$$A(q_0, \gamma_0^G, \gamma_1^G) \equiv \frac{\gamma_0^G}{\beta\Big((1-\rho) + \rho\Omega(q_0, \gamma_1^G)\Big)} - 1 - \sigma\theta \frac{u'(q_0) - c'(q_0)}{\theta c'(q_0) + (1-\theta)u'(q_0)},$$

where

$$\Omega(q_0, \gamma_1^G) \equiv (1-\rho)\,\frac{\omega(q_0, \gamma_1^G)}{1 - \rho\,\omega(q_0, \gamma_1^G)}, \qquad \omega(q_0, \gamma_1^G) \equiv \frac{z_\theta(q_1(\gamma_1^G))}{z_\theta(q_0)}.$$

Note that $\omega_q' \equiv \frac{\partial \omega(q_0, \gamma_1^G)}{\partial q_0} < 0$, $\omega_\gamma' \equiv \frac{\partial \omega(q_0, \gamma_1^G)}{\partial \gamma_1^G} < 0$ and

$$\frac{\partial \Omega(q_0, \gamma_1^G)}{\partial q_0} = \frac{(1-\rho)\omega_q'(1-\rho\omega) - (1-\rho)\omega\rho(-\omega_q')}{\Big(1-\rho\omega\Big)^2} < 0,$$

$$\frac{\partial \Omega(q_0, \gamma_1^G)}{\partial \gamma_1^G} = \frac{(1-\rho)\omega_\gamma'(1-\rho\omega) - (1-\rho)\omega\rho(-\omega_\gamma')}{\Big(1-\rho\omega\Big)^2} < 0.$$

Therefore:

$$\frac{\partial A(q_0, \gamma_0^G, \gamma_1^G)}{\partial q_0} = \frac{\gamma_0^G}{\beta\Big((1-\rho) + \rho\Omega(q_0, \gamma_1^G)\Big)^2} \overbrace{(-\rho)\frac{\partial \Omega(q_0, \gamma_1^G)}{\partial q_0}}^{>0}$$

$$-\underbrace{\frac{\sigma\theta\Big(u''(q_0)-c''(q_0)\Big)\Big(\theta c'(q_0)+(1-\theta)u'(q_0)\Big) \\ -\sigma\theta\Big(u'(q_0)-c'(q_0)\Big)\Big(\theta c''(q_0)+(1-\theta)u''(q_0)\Big)}{\Big(\theta c'(q_0)+(1-\theta)u'(q_0)\Big)^2}}_{<0} > 0,$$

$$\frac{\partial A(q_0,\gamma_0^G,\gamma_1^G)}{\partial\gamma_0^G} = \frac{1}{\beta\Big((1-\rho)+\rho\Omega(q_0,\gamma_1^G)\Big)} > 0,$$

$$\frac{\partial A(q_0,\gamma_0^G,\gamma_1^G)}{\partial\gamma_1^G} = \frac{\gamma_0^G}{\beta\Big((1-\rho)+\rho\Omega(q_0,\gamma_1^G)\Big)^2}(-\rho)\overbrace{\frac{\partial\Omega(q_0,\gamma_1^G)}{\partial\gamma_1^G}}^{>0} > 0.$$

By definition of $q_0(\gamma_0^G,\gamma_1^G)$ it holds that $A(q_0(\gamma_0^G,\gamma_1^G),\gamma_0^G,\gamma_1^G) = 0$. Then, by the implicit function theorem it follows that:

$$\frac{\partial q_0(\gamma_0^G,\gamma_1^G)}{\partial\gamma_0^G} = -\frac{\frac{\partial A(q_0,\gamma_0^G,\gamma_1^G)}{\partial\gamma_0^G}}{\frac{\partial A(q_0,\gamma_0^G,\gamma_1^G)}{\partial q_0}} < 0,$$

$$\frac{\partial q_0(\gamma_0^G,\gamma_1^G)}{\partial\gamma_1^G} = -\frac{\frac{\partial A(q_0,\gamma_0^G,\gamma_1^G)}{\partial\gamma_1^G}}{\frac{\partial A(q_0,\gamma_0^G,\gamma_1^G)}{\partial q_0}} < 0.$$

Moreover, notice that $0 < q_0(\gamma_0^G,\gamma_1^G) < q^*$ since:

$$A(0,\gamma_0^G,\gamma_1^G) = -1-\sigma\frac{\theta}{1-\theta} < 0,$$

$$A(q^*,\gamma_0^G,\gamma_1^G) = \underbrace{\frac{\gamma_0^G}{\beta\Big((1-\rho)+\rho\Omega(q^*,\gamma_1^G)\Big)}}_{>1} -1 > 0,$$

for every $\gamma_0^G \geq 1$ and $\bar{\gamma}^G > \gamma_1^G \geq 1$ where in the latter we have used $u'(q^*) = c'(q^*)$. Since A is continuous and strictly increasing in q_0, with $A(0,\cdot) < 0$ and $A(q^*,\cdot) > 0$, there exists a unique $q_0 \in (0,q^*)$ solving $A = 0$. ■

D.3 Proof Lemma 3

Lemma 3. *Given (γ_1^G,γ^C), the buyer's post-event demand for currency (z_1^G,z_1^B) and DM-consumption q_1 is given by:*

- *If $\gamma^C \geq \gamma_1^G$ then $q_1 = \mathcal{L}^{-1}\Big(\frac{\gamma_1^G}{\beta}-1\Big)$, $z_1^G = z_\theta(q_1)$ and $z_1^C = 0$,*
- *If $\gamma^C < \gamma_1^G \leq \hat{\gamma}(\gamma^C)$ then $q_1 = \mathcal{L}^{-1}\Big(\frac{\gamma_1^G}{\beta}-1\Big)$, $z_1^G = z_\theta(q_1) - z^C + \theta\varphi(D^{-1}(z^C)) \geq 0$ where $z_1^C > 0$ solves $\frac{\gamma^C}{\beta} = 1+\mathcal{L}(q_1)-\mathcal{T}(q_1,z_1^C)$ such that $\frac{\partial z_1^C}{\partial\gamma_1^G} > 0$,*
- *If $\gamma_1^G > \hat{\gamma}$ then $z_1^G = 0$ and $z_1^C = \hat{z}(\gamma^C) > 0$ solves $\frac{\gamma^C}{\beta} = 1+\mathcal{L}(q_1)-\mathcal{T}(q_1,z_1^C)$ where $q_1 = z_\theta^{-1}\Big(z_1^C - \theta\varphi(D^{-1}(z_1^C))\Big)$,*

where $\hat{\gamma}(\gamma^C)$ and $\hat{z}(\gamma^C)$ jointly uniquely solve:

$$z_\theta\Big(\mathcal{L}^{-1}(\frac{\hat{\gamma}}{\beta}-1)\Big) = \hat{z} - \theta\varphi\Big(D^{-1}(\hat{z})\Big), \tag{D.5}$$

$$\hat{\gamma} - \gamma^C = \beta\mathcal{T}\Big(\mathcal{L}^{-1}(\frac{\hat{\gamma}}{\beta}-1), \hat{z}\Big), \tag{D.6}$$

such that $\hat{\gamma} > \gamma^C$ and the system has a unique solution (see Appendix D.4).

Proof. Given that $z_1^G > 0$, $z_1^C > 0$, or both, the FOC w.r.t. cash holds with equality, the FOC w.r.t. crypto holds with equality, or both do (see (25)).

Case 1: $z_1^G > 0$ and $z_1^C = 0$: The first equation in (25) holds with equality while the second with inequality. Evaluating (25) at $z_1^C = 0$ immediately implies $\gamma_1^C \geq \gamma_1^G$. Since $\mathcal{L}(q_1)$ is strictly decreasing for any $q < q^*$ and $\lim_{q\to 0}\mathcal{L}(q) \to \infty$ and $\lim_{q\to q^*}\mathcal{L}(q) \to 0$, it follows that there always exists a unique $0 < q < q^*$ such that $1 < \gamma^G/\beta = 1 + \mathcal{L}(q_1)$. Hence, by (25), it follows that $q_1 = \mathcal{L}^{-1}\Big(\frac{\gamma^G}{\beta} - 1\Big)$, $z_1^G = z_\theta(q_1)$ and $z_1^C = 0$ if $\gamma_1^C \geq \gamma_1^G$.

Case 2: $z_1^G > 0$ and $z_1^C > 0$: Both equations in (25) hold with equality. (q_1, z_1^G, z_1^C) then jointly solve:

$$q_1 = \mathcal{L}^{-1}\Big(\frac{\gamma^G}{\beta} - 1\Big), \tag{D.7}$$

$$\Gamma(q_1, z_1^C) \equiv \frac{\gamma_1^C}{\beta} - 1 - \mathcal{L}(q_1) + \mathcal{T}(q_1, z_1^C) = 0, \tag{D.8}$$

$$z_1^G = z_\theta(q_1) - z^C + \theta\varphi(D^{-1}(z^C)). \tag{D.9}$$

First, as established above, (D.7) has a unique solution $q_1(\gamma^G)$ given $\gamma_1^G < \bar{\gamma}$.

Second, we show under what conditions a unique solution to (D.8) exists. Note that $\lim_{z\to\infty}\Gamma(q_1, z) = \infty$ and, assuming $\gamma^C < \gamma^G$, also $\Gamma(q_1, 0) < 0$ (if $\gamma^C \geq \gamma^G$, then $\Gamma(q_1, 0) \geq 0$).[38] Moreover, since $\Gamma(q_1, z)$ is strictly increasing in z, there exists a unique $z^C(q_1)$ whenever $\gamma^C < \gamma^G$; otherwise, no solution exists.

Third, we show that (D.8) implies $\partial z^C(q_1(\gamma_1^G))/\partial\gamma_1^G > 0$. By the implicit function theorem, this holds since $\frac{\partial z_1^C}{\partial \gamma_1^G} = -\frac{\frac{\partial\Gamma(q_1,z_1^C)}{\partial q_1}\frac{\partial q_1}{\partial\gamma_1^G}}{\frac{\partial\Gamma(q_1,z_1^C)}{\partial z_1^C}}$. Specifically:

$$\begin{aligned}
\frac{\partial\Gamma(q_1, z_1^C)}{\partial q_1} &= \partial\Big(-\mathcal{L}(q_1) + \mathcal{T}(q_1, z_1^C)\Big)\Big/\partial q_1 \\
&= \partial\Big(\sigma\theta\frac{u'(q_1) - c'(q_1)\Big(1+\varphi'(d^C)\Big)}{\Big(1+\varphi'(d^C)\Big)\Big(\theta c'(q) + (1-\theta)u'(q)\Big)}\Big)\Big/\partial q_1 \\
&= \sigma\theta\frac{\Big(\theta c'(q) + (1-\theta)u'(q)\Big)\Big(u''(q) - c''(q)(1+\varphi'(d^C))\Big)}{\Big(1+\varphi'(d^C)\Big)\Big(\theta c'(q) + (1-\theta)u'(q)\Big)^2}
\end{aligned}$$

[38] We use that $\mathcal{T}(q_1, 0) = 0$, $\lim_{z^C\to\infty}\mathcal{T}(q_1, z^C) = \infty$, and $\mathcal{T}(q_1, z^C)$ is strictly increasing in z^C for some $0 < q_1 < q^*$.

$$- \sigma\theta \frac{\Big(\theta c''(q) + (1-\theta)u''(q)\Big)\Big(u'(q) - c'(q)[1+\varphi'(d^C)]\Big)}{\Big(1+\varphi'(d^C)\Big)\Big(\theta c'(q) + (1-\theta)u'(q)\Big)^2},$$

$$= \sigma\theta \frac{\Big(1+\varphi'(d^C)(1-\theta)\Big)\Big[u''(q)c'(q) - c''(q)u'(q)\Big]}{\Big(1+\varphi'(d^C)\Big)\Big(\theta c'(q) + (1-\theta)u'(q)\Big)^2} < 0.$$

Moreover, since $\partial\Gamma(q_1, z_1^C)/\partial z_1^C < 0$, given $\partial\mathcal{T}(q,z)/\partial z > 0$, and $\partial q_1/\partial\gamma_1^G < 0$, as shown above, it follows that $\partial z_1^C/\partial\gamma_1^G > 0$.

Finally, we derive conditions under which a unique solution to (D.9) exists. Observe that z_1^G as implied by (D.9) always uniquely exists but is strictly positive only when $\gamma^G < \hat{\gamma}$. To see this, note that $\gamma_1^G \to \gamma^C$ implies, from (D.7) and (D.8), that $z^C \to 0$. Moreover, since $\partial z_1^C/\partial\gamma_1^G > 0$ and $\partial q_1/\partial\gamma_1^G < 0$, it follows from (D.9) that there exists some $\hat{\gamma} > \gamma^C$ and $\hat{z} > 0$ such that $z_1^C = \hat{z}$ and $z_1^G = 0$; the uniqueness of $(\hat{\gamma}, \hat{z})$ as defined by (D.5) and (D.6) is established in Appendix D.4.[39]

To conclude, $z_1^G > 0$ and $z_1^C > 0$ if $\gamma^C < \gamma_1^G < \hat{\gamma}$.

Case 3: $z_1^G = 0$ **and** $z_1^C > 0$**:** The first equation in (25) holds with inequality and the second with equality. For this to hold, it immediately follows that $\gamma^G > \hat{\gamma}$ (see definition of $\hat{\gamma}$). Then, using similar arguments as in Case 2, (q_1, z_1^C) jointly and uniquely solve

$$\gamma_1^C/\beta = 1 + \mathcal{L}(q_1) - \mathcal{T}(q_1, z_1^C) \quad \text{and} \quad z^C - \theta\varphi(D^{-1}(z^C)) = z_\theta(q_1).$$

■

D.4 Uniqueness of $(\hat{\gamma}, \hat{z})$

Proof. Define $\hat{z}^{(1)}(\hat{\gamma})$ and $\hat{z}^{(2)}(\hat{\gamma})$ solving respectively:

$$\Lambda^{(1)}\Big(\hat{\gamma}, \hat{z}^{(1)}(\hat{\gamma})\Big) \equiv z_\theta\Big(\mathcal{L}^{-1}(\frac{\hat{\gamma}}{\beta} - 1)\Big) - \hat{z} + \theta\varphi(D^{-1}(\hat{z})) = 0,$$

$$\Lambda^{(2)}\Big(\hat{\gamma}, \hat{z}^{(2)}(\hat{\gamma})\Big) \equiv \hat{\gamma} - \gamma^C - \beta\mathcal{T}(\mathcal{L}^{-1}(\frac{\hat{\gamma}}{\beta} - 1), \hat{z}) = 0.$$

Sufficient conditions for uniqueness are $\frac{\partial\hat{z}^{(1)}(\hat{\gamma})}{\partial\hat{\gamma}} < 0$ and $\frac{\partial\hat{z}^{(2)}(\hat{\gamma})}{\partial\hat{\gamma}} > 0$. Both hold true:

1. By the implicit function theorem $\frac{\partial\hat{z}^{(1)}(\hat{\gamma})}{\partial\hat{\gamma}} = -\frac{\partial\Lambda^{(1)}(\hat{\gamma},\hat{z})}{\partial\hat{\gamma}}\Big/\frac{\partial\Lambda^{(1)}(\hat{\gamma},\hat{z})}{\partial\hat{z}}$ <0 since

$$\frac{\partial\Lambda^{(1)}(\hat{\gamma},\hat{z})}{\partial\hat{\gamma}} = \frac{z'_\theta(q)}{\beta\mathcal{L}'(q)} < 0, \quad \frac{\partial\Lambda^{(1)}(\hat{\gamma},\hat{z})}{\partial\hat{z}} = -\frac{1+(1-\theta)\varphi'(d^C)}{1+\varphi'(d^C)} < 0.$$

2. By the implicit function theorem $\frac{\partial\hat{z}^{(2)}(\hat{\gamma})}{\partial\hat{\gamma}} = -\frac{\partial\Lambda^{(2)}(\hat{\gamma},\hat{z})}{\partial\hat{\gamma}}\Big/\frac{\partial\Lambda^{(2)}(\hat{\gamma},\hat{z})}{\partial\hat{z}}$ >0 since:

$$\frac{\partial\Lambda^{(2)}(\hat{\gamma},\hat{z})}{\partial\hat{\gamma}} = 1 - \frac{\frac{\partial\mathcal{T}(q,\hat{z})}{\partial q}}{\mathcal{L}'(q)}$$

[39]It is easy to see that $\hat{\gamma} < \bar{\gamma}$.

$$= 1 - \underbrace{\frac{\theta\Big[u''(q)c'(q) - u'(q)c''(q)\Big]}{\theta\Big[u''(q)c'(q) - u'(q)c''(q)\Big] + (1-\theta)\Big[u''(q)c'(q) - u'(q)c''(q)\Big]}}_{=\theta<1} \cdot \underbrace{\frac{\varphi'(d^C)}{1+\varphi'(d^C)}}_{<1} > 0,$$

$$\frac{\partial \Lambda^{(2)}(\hat{\gamma}, \hat{z})}{\partial \hat{z}} = -\beta \frac{\partial \mathcal{T}(q, \hat{z})}{\partial \hat{z}} < 0.$$

■

D.5 Proof Proposition 1

Notation-wise, we denote by $R(\gamma_1^G \mid \delta)$ the seigniorage revenue function as a function of the adoption rate $\delta \in \{0, 1\}$, and by $\gamma_1^{G,*}(\delta)$ the optimal post-event money-growth rate chosen by the government that maximizes $R(\gamma_1^G \mid \delta)$.

Proof. We first prove that $\gamma^C \leq \gamma_1^{G,*}(1)$, then $\gamma_1^{G,*}(1) < \hat{\gamma}$ and finally $\gamma_1^{G,*}(1) < \gamma_1^{G,*}(0)$.

Step 1: $\gamma^C \leq \gamma_1^{G,*}(1)$: Suppose not. Then $\gamma^C > \gamma_1^{G,*}(1)$ and therefore, by definition of $\gamma_1^{G,*}(1)$, it holds that $R(\gamma^C \mid 1) < R(\gamma_1^{G,*} \mid 1)$ given that R has a unique maximum.[40] Since, by assumption, $\gamma^C < \gamma_1^{G,*}(0)$, and $R(\gamma_1^G \mid \delta)$ is strictly increasing for all $\gamma_1^G < \gamma_1^{G,*}$ it follows that $R(\gamma_1^G \mid 0) < R(\gamma^C \mid 0) < R(\gamma_1^{G,*}(0) \mid 0)$ for all $\gamma_1^G < \gamma^C$. But note that, by Lemma 3, $R(\gamma_1^G \mid 1) = R(\gamma_1^G \mid 0)$ for all $\gamma_1^G < \gamma^C$ which implies that $R(\gamma_1^G \mid 1) < R(\gamma^C \mid 1)$ for all $\gamma_1^G < \gamma^C$. But since, by assumption, $\gamma^C > \gamma_1^{G,*}(1)$ it implies that $R(\gamma_1^{G,*}(1) \mid 1) < R(\gamma^C \mid 1)$. A contradiction.

Step 2: $\gamma_1^{G,*}(1) < \hat{\gamma}$: Suppose not. Then $\gamma_1^{G,*}(1) \geq \hat{\gamma}$ and, by Lemma 3, $Z_1^G = 0$. This is can not be optimal given that there always exists some γ^G such that $R(\gamma^G \mid 1) > 0$.

Step 3: $\gamma_1^{G,*}(1) < \gamma_1^{G,*}(0)$: The FOC of the government is given by

$$\gamma_1^{G,*}(\delta) = 1 + \frac{Z_1^G(\gamma^{G,*} \mid \delta)}{-\dfrac{\partial Z_1^G(\gamma^{G,*} \mid \delta)}{\partial \gamma^G}}. \tag{D.10}$$

Sufficient conditions that $\gamma^{G,*}(1) < \gamma^{G,*}(0)$ are $Z_1^G(\gamma \mid 1) < Z_1^G(\gamma \mid 0)$ and $-\dfrac{\partial Z_1^G(\gamma^{G,*} \mid 1)}{\partial \gamma^G} > -\dfrac{\partial Z_1^G(\gamma^{G,*} \mid 0)}{\partial \gamma^G}$. Both are true:

1. Since, as shown above, $\gamma_1^{G,*}(1) \in \Big[\gamma^C, \hat{\gamma}\Big)$ and, by Lemma 3, $Z_1^G = z_\theta(q_1) - z^C + \theta\varphi(D^{-1}(z^C))$ when $\delta = 1$, whereas $Z_1^G = z_\theta(q_1)$ when $\delta = 0$. Since $-z^C + \theta\varphi(D^{-1}(z^C)) < 0$ it follows that $Z_1^G(\gamma \mid 1) < Z_1^G(\gamma \mid 0)$.

[40] A sufficient condition is that $Z_1^{G\prime}(\gamma_1^G) < 0$ and $Z_1^{G\prime\prime}(\gamma_1^G) < 0$ for all $\gamma_1^G > 1$; both are straightforward to verify.

2. It holds that:

$$\frac{\partial Z_1^G(\gamma^{G,*}\mid 0)}{\partial \gamma^G}=\frac{z_\theta'(q_1)}{\beta\mathcal{L}'(q)},\quad \frac{\partial Z_1^G(\gamma^{G,*}\mid 1)}{\partial \gamma^G}=\frac{z_\theta'(q_1)}{\beta\mathcal{L}'(q)}-\underbrace{\frac{\partial z_1^C}{\partial \gamma_1^G}\Big[1-\theta\frac{\varphi'(d^C)}{1+\varphi'(d^C)}\Big]}_{>0} \tag{D.11}$$

where we have used $\frac{\partial z_1^C}{\partial \gamma_1^G}>0$ from Lemma 3. This then implies that $-\frac{\partial Z_1^G(\gamma^{G,*}\mid 1)}{\partial \gamma^G}>-\frac{\partial Z_1^G(\gamma^{G,*}\mid 0)}{\partial \gamma^G}$.

■

D.6 Proof Proposition 3

Proof. A partial-adoption equilibrium are values (δ_1, z_1^G, z_1^C) that satisfy:

$$\chi=\sigma(1-\theta)\Big(\mathcal{S}(z_1^G,z_1^C)-\mathcal{S}(z_1^G,0)\Big),$$

$$\frac{\gamma_1^G}{\beta}=1+\sigma\theta\Big[\delta_1\frac{\partial\mathcal{S}(z_1^G,z_1^C)}{\partial z_1^G}+(1-\delta_1)\frac{\partial\mathcal{S}(z_1^G,0)}{\partial z_1^G}\Big],$$

$$\frac{\gamma_1^C}{\beta}=1+\sigma\theta\delta_1\frac{\partial\mathcal{S}(z_1^G,z_1^C)}{\partial z_1^C}.$$

Totally differentiate (C.1)-(C.3) with respect to the exogenous variable γ^G and the endogenous variables (δ, z^C, z^G) (we drop s-subscript for readability):

$$0=dz^G\Big(\mathcal{S}_G-\tilde{\mathcal{S}}_G\Big)+dz^C\cdot\mathcal{S}_C, \tag{D.12}$$

$$\frac{d\gamma^G}{\beta\theta\sigma}=dz^G\Big(\delta\mathcal{S}_{GG}+(1-\delta)\tilde{\mathcal{S}}_{GG}\Big)+dz^C\cdot\delta\mathcal{S}_{GC}+d\delta\Big(\mathcal{S}_G-\tilde{\mathcal{S}}_G\Big), \tag{D.13}$$

$$0=dz^G\cdot\delta\mathcal{S}_{CG}+dz^C\cdot\delta\mathcal{S}_{CC}+d\delta\cdot\mathcal{S}_C, \tag{D.14}$$

where $\mathcal{S}_c$ denotes the partial derivative of $\mathcal{S}(z^G,z^C)$ with respect to $z^c\in\{z^G,z^C\}$, and $\mathcal{S}_{cc'}$ denotes the second partial derivative with respect to z^c and $z^{c'}\in\{z^G,z^C\}$. Derivatives of $\mathcal{S}(z^G,0)$ are analogously denoted by $\tilde{\mathcal{S}}_c$ and $\tilde{\mathcal{S}}_{cc'}$.

Dividing equation (D.12) by $d\gamma^G$ and rewriting yields:

$$\frac{dz^C}{d\gamma^G}=\underbrace{\Big(\frac{-\mathcal{S}_C}{\mathcal{S}_G-\tilde{\mathcal{S}}_G}\Big)}_{>0}\frac{dz^G}{d\gamma^G}, \tag{D.15}$$

which implies that if $\frac{dz^G}{d\gamma^G}\geq(<)0$ then $\frac{dz^C}{d\gamma^G}\geq(<)0$. Then dividing equation (D.14) by $d\gamma^G$ and rewriting yields:

$$\frac{d\delta}{d\gamma^G}=\delta\frac{(-\mathcal{S}_{CG}\frac{dz^G}{d\gamma^G})+(-\mathcal{S}_{CC}\frac{dz^C}{d\gamma^G})}{\mathcal{S}_C}. \tag{D.16}$$

It follows that, given that $-\mathcal{S}_{CG}>0$, $-\mathcal{S}_{CC}>0$ and $\mathcal{S}_C>0$, if $\frac{dz^C}{d\gamma^G}\geq(<)0$ then $\frac{dz^G}{d\gamma^G}\geq(<)0$, by (D.15), and $\frac{d\delta}{d\gamma^G}\geq(<)0$, by (D.16). Finally, we show that $\frac{dz^C}{d\gamma^G}<0$. Suppose not. Then $\frac{dz^C}{d\gamma^G}\geq 0$. But then equation (D.13) after dividing by $d\gamma^G$ reads:

$$0 < \frac{1}{\beta\theta\sigma} = \frac{dz^G}{d\gamma^G} \underbrace{\Big(\delta\mathcal{S}_{GG} + (1-\delta)\tilde{\mathcal{S}}_{GG}\Big)}_{<0} + \frac{dz^C}{d\gamma^G}\delta\underbrace{\mathcal{S}_{GC}}_{<0} + \frac{d\delta}{d\gamma^G}\underbrace{\Big(\mathcal{S}_G - \tilde{\mathcal{S}}_G\Big)}_{<0} < 0,$$

where the right-hand side is negative because if $\frac{dz^C}{d\gamma^G} \geq 0$, by assumption, then $\frac{dz^G}{d\gamma^G} \geq 0$, by (D.15), and $\frac{d\delta}{d\gamma^G} \geq 0$, by (D.16). A contradiction with the positive left-hand side. Therefore, $\frac{dz^C}{d\gamma^G} < 0$, $\frac{dz^G}{d\gamma^G} < 0$ and $\frac{d\delta}{d\gamma^G} < 0$.

Then similarly, totally differentiate (C.1)-(C.3) with respect to the exogenous variable γ^C and the endogenous variables (δ, z^C, z^G):

$$0 = dz^G\Big(\mathcal{S}_G - \tilde{\mathcal{S}}_G\Big) + dz^C \cdot \mathcal{S}_C, \tag{D.17}$$

$$0 = dz^G\Big(\delta\mathcal{S}_{GG} + (1-\delta)\tilde{\mathcal{S}}_{GG}\Big) + dz^C \cdot \delta\mathcal{S}_{GC} + d\delta\Big(\mathcal{S}_G - \tilde{\mathcal{S}}_G\Big), \tag{D.18}$$

$$\frac{d\gamma^C}{\beta\sigma\theta} = dz^G \cdot \delta\mathcal{S}_{CG} + dz^C \cdot \delta\mathcal{S}_{CC} + d\delta \cdot \mathcal{S}_C. \tag{D.19}$$

Identical to before, (D.17) implies that if $\frac{dz^C}{d\gamma^C} \geq (<)0$ then $\frac{dz^G}{d\gamma^C} \geq (<)0$. Dividing by $d\gamma^C$ and rewriting equation (D.18) yields:

$$\frac{d\delta}{d\gamma^C} = \frac{\frac{dz^G}{d\gamma^C}\overbrace{\Big(-\delta\mathcal{S}_{GG} - (1-\delta)\tilde{\mathcal{S}}_{GG}\Big)}^{>0} + \frac{dz^C}{d\gamma^C}\cdot\overbrace{(-\delta\mathcal{S}_{GC})}^{>0}}{\underbrace{\Big(\mathcal{S}_G - \tilde{\mathcal{S}}_G\Big)}_{<0}}. \tag{D.20}$$

Therefore, if $\frac{dz^C}{d\gamma^C} \geq (<)0$ then $\frac{dz^G}{d\gamma^C} \geq (<)0$, by (D.17), and $\frac{d\delta}{d\gamma^C} \leq (>)0$, by (D.20). Finally, we show that $\frac{dz^C}{d\gamma^C} < 0$. Suppose not. Then $\frac{dz^C}{d\gamma^C} \geq 0$. Then dividing equation (D.19) by $d\gamma^C$:

$$0 < \frac{1}{\beta\sigma\theta} = \frac{dz^G}{d\gamma^C}\cdot\underbrace{\delta\mathcal{S}_{CG}}_{<0} + \frac{dz^C}{d\gamma^C}\cdot\underbrace{\delta\mathcal{S}_{CC}}_{<0} + \frac{d\delta}{d\gamma^C}\cdot\underbrace{\mathcal{S}_C}_{>0} < 0,$$

where the right-hand side is negative because if $\frac{dz^C}{d\gamma^C} \geq 0$, by assumption, then $\frac{dz^G}{d\gamma^C} \geq 0$, by (D.17), and $\frac{d\delta}{d\gamma^C} \leq 0$, by (D.20). A contradiction with the positive left-hand side. Therefore, $\frac{dz^C}{d\gamma^C} < 0$, $\frac{dz^G}{d\gamma^C} < 0$ and $\frac{d\delta}{d\gamma^C} > 0$. ■

D.7 Derivation of the Welfare Function (27)

Welfare is given by $\mathcal{W}(\delta) \equiv V_0^B(\delta) + V_0^S(\delta) + V_0^G(\delta)$, where $\delta \in \{0,1\}$ and all value functions are evaluated at equilibrium real balances. Since post-event government consumption $\tilde{x}_1$ cancels with government expenditure g_1, we redefine $V_0^B(\delta)$ and $V_0^S(\delta)$ to exclude government expenditure post-event, so that $\mathcal{W}(\delta) = V_0^B(\delta) + V_0^S(\delta)$. Inserting the value functions:

$$\mathcal{W}(\delta) = \sigma\mathcal{S}_0(\delta) + (1-\rho)W_0(\delta) + \rho W_1(\delta), \tag{D.21}$$

where $W_s(\delta) \equiv W_s^B(\delta) + W_s^S(\delta)$ for $s = \{0, 1\}$. Note that $W_0(\delta)$ and $W_0(\delta)$ can be written as:

$$W_0(\delta) = 2 \cdot \Big(U(x^*) - x^*\Big) + 2v(g^*) - g^* + \beta \mathcal{W}(\delta), \tag{D.22}$$

$$W_1(\delta) = 2 \cdot \Big(U(x^*) - x^*\Big) - \delta\chi + \beta V_1(\delta), \tag{D.23}$$

where $V_s(\delta) \equiv V_s^B(\delta) + V_s^S(\delta)$. Note that all z-terms cancel. Inserting (D.22) and (D.23) into (D.21) and solving for $\mathcal{W}(\delta)$ yields:

$$\mathcal{W}(\delta) = \frac{\sigma \mathcal{S}_0(\delta) + 2\big(U(x^*) - x^*\big) + (1-\rho)\Big(2v(g^*) - g^*\Big) - \rho\delta\chi + \rho\beta V_1(\delta)}{1 - \beta(1-\rho)}. \tag{D.24}$$

Then $V_1(\delta)$ can be written as:

$$V_1(\delta) = \sigma \mathcal{S}_1(\delta) + W_1(\delta) = \frac{\sigma \mathcal{S}_1(\delta) + 2 \cdot \Big(U(x^*) - x^*\Big) - \chi}{1 - \beta}. \tag{D.25}$$

The welfare function can therefore be written as:

$$\begin{aligned} \mathcal{W}(\delta) = {} & \frac{\sigma}{1 - \beta(1-\rho)} \mathcal{S}_0(\delta) + \frac{\rho\beta\sigma}{\big(1 - \beta(1-\rho)\big)(1-\beta)} \mathcal{S}_1(\delta) \\ & + \frac{2}{1-\beta}\big(U(x^*) - x^*\big) + \frac{1-\rho}{1-\beta(1-\rho)}\big(2v(g^*) - g^*\big) \\ & - \frac{\rho}{\big(1-\beta(1-\rho)\big)(1-\beta)} \delta\chi. \end{aligned} \tag{D.26}$$